

# Logic Testing and Design for Testability

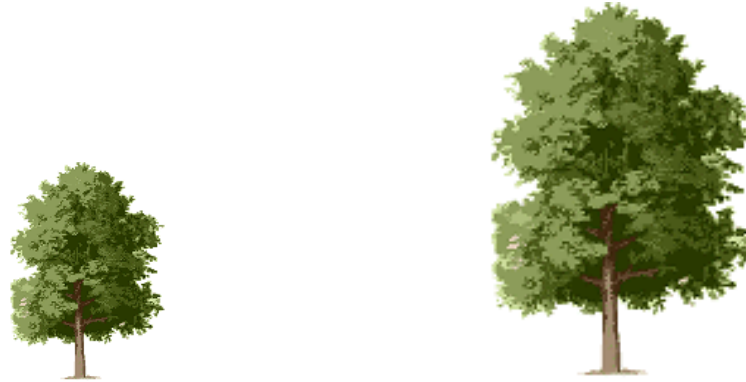
MIT Press, Sept. 1985



Hideo Fujiwara

# The basis is necessary for development

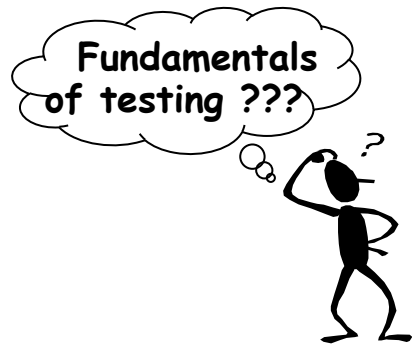
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- ❑ For a tree to grow larger, its root must grow bigger and deeper into the ground.
- ❑ Similarly, for test technologies (leaves) to develop, substantial results of the fundamental research (root) are necessary.
- ❑ The more enriched the **fundamental** research results become, the more enriched the **practical** research results become.

# Fundamental problems of testing

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What are the fundamentals of testing?

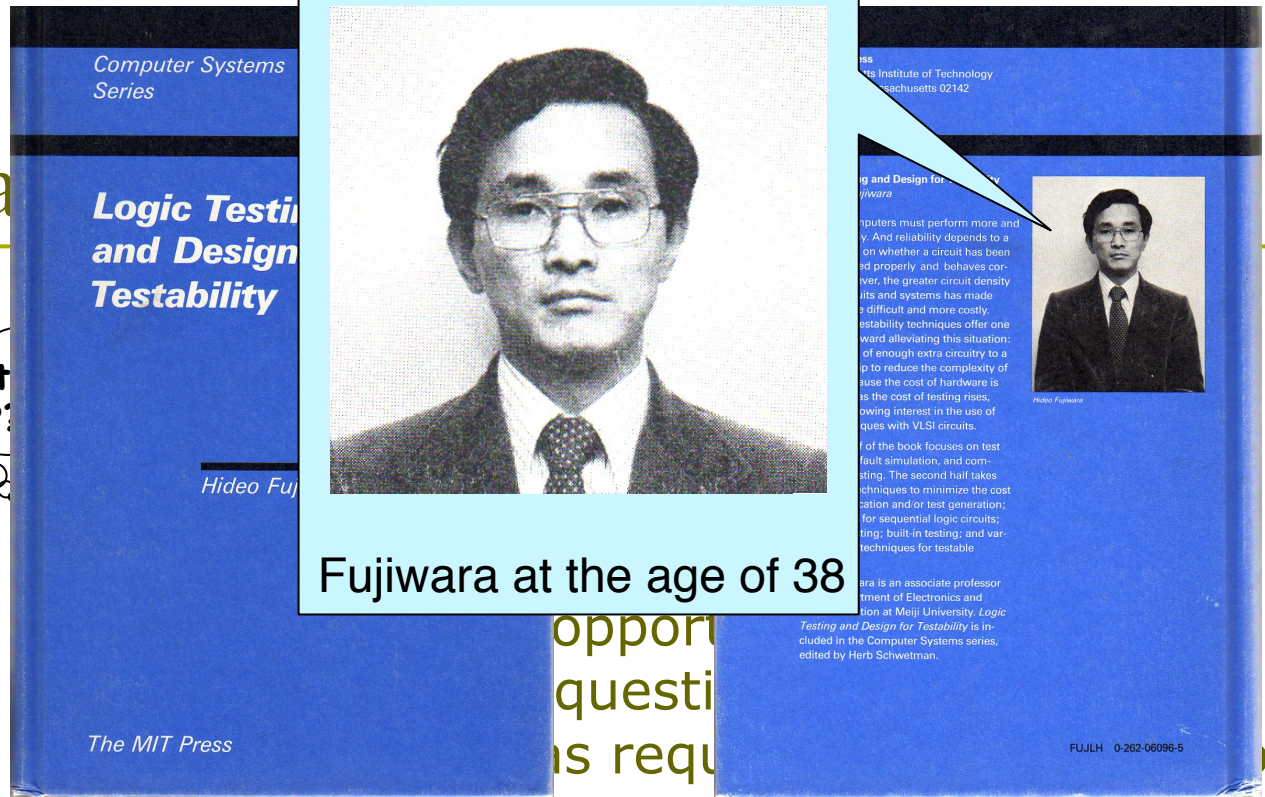
I had the opportunity to ask myself the same question when I was requested to write a book.

To educate the fundamentals of testing, I wrote a book.

Hideo Fujiwara, *Logic Testing and Design for Testability*,  
The MIT Press, 1985

# Funda

Fundamentals of testing??

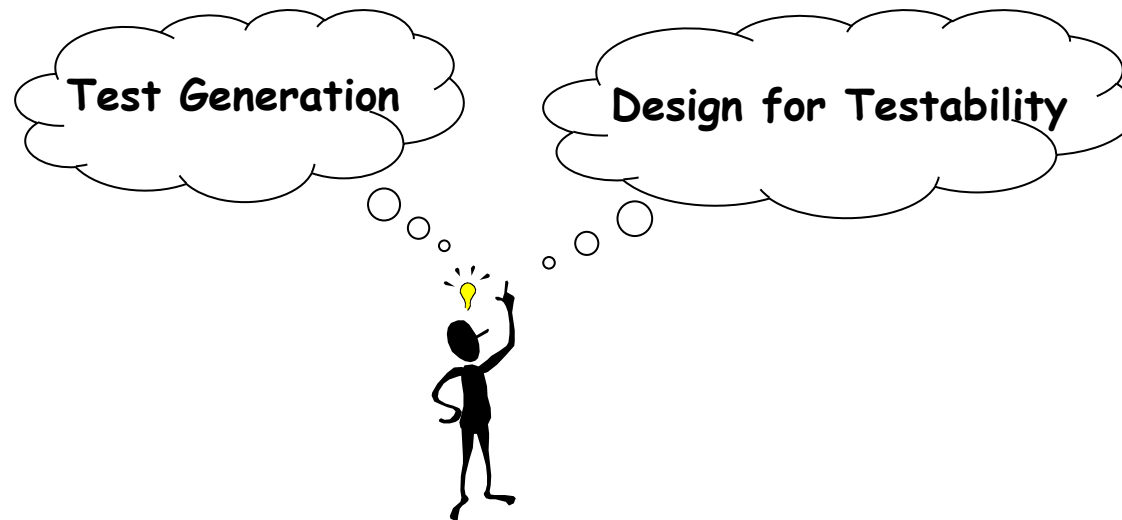


Fujiwara at the age of 38

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Hideo Fujiwara, *Logic Testing and Design for Testability*,  
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# Fundamental problems of testing

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- Test generation problem
  - ATPG (Automatic Test Program Generation)
  
- Design-for-test problem
  - DFT



# H. Fujiwara, *Logic Testing and Design for Testability*, MIT Press, 1985

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## **I LOGIC TESTING**

### **1 Introduction to Logic Testing**

- 1.1 Logic Circuits
- 1.2 Fault Modeling
- 1.3 Testing Problems
- 1.4 Testing Schemes

### **2 Test Generation**

- 2.1 Boolean Difference
- 2.2 The D-Algorithm
- 2.3 The PODEM Algorithm
- 2.4 The FAN Algorithm
- 2.5 Test Generation for Sequential Circuits

### **3 Fault Simulation**

- 3.1 Simulation Methodology
- 3.2 Parallel Fault Simulation
- 3.3 Deductive Fault Simulation
- 3.4 Concurrent Fault Simulation
- 3.5 Hardware Simulators

### **4 The Complexity of Testing**

- 4.1 NP-Completeness
- 4.2 Polynomial Time Class
- 4.3 Closedness under Faults

## **II DESIGN FOR TESTABILITY**

### **5 Introduction to Design for Testability**

- 5.1 Testability
- 5.2 Minimization of Testing Cost
- 5.3 Combinational Logic versus Sequential Logic
- 5.4 Ad Hoc Design and Structured Design

### **6 Design to Minimize the Cost of Test Application**

- 6.1 EOR Embedding
- 6.2 Minimally Testable Design
- 6.3 Dual-Mode Logic
- 6.4 Testing with Fixed Reference Values

### **7 Design to Minimize the Cost of Test Generation**

- 7.1 Partitioning and Exhaustive Testing
- 7.2 Syndrome-Testable Design
- 7.3 Reed-Muller Canonical Forms
- 7.4 Programmable Logic Arrays

### **8 Scan Design for Sequential Logic Circuits**

- 8.1 State-Shiftable Machines
- 8.2 Scan Design Approaches
- 8.3 Variations of Scan Design
- 8.4 Incomplete Scan Design and Enhanced Scan Design

### **9 Design for Built-in Self-Testing**

- 9.1 Signature Analysis
- 9.2 Built-In Logic Block Observer
- 9.3 Self-Test with Scan Design
- 9.4 Self-Verification

# H. Fujiwara, *Logic Testing and Design for Testability*, MIT Press, 1985

## Practical items

- 1.1 Logic Circuits
- 1.2 Fault Models
- 1.3 Testing Procedures
- 1.4 Testing Schemes

### Test Generation

## 2 Test Generation

- 2.1 Boolean Difference
- 2.2 The D-Algorithm
- 2.3 The PODEM
- 2.4 The FAN Algorithm
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### Fault Simulation

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Theoretical items

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Boolean Difference

The Complexity of Testing

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Design to Minimize the Cost of Test Application

Design to Minimize the Cost of Test Generation



# Fundamental problems of testing

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## Practical synthesis problems

- Test generation algorithms.
  - Invent efficient algorithms to provide high fault efficiency.
- Design-for-testability methods.
  - Optimize the DFT under various constraints.

## Theoretical analysis problems

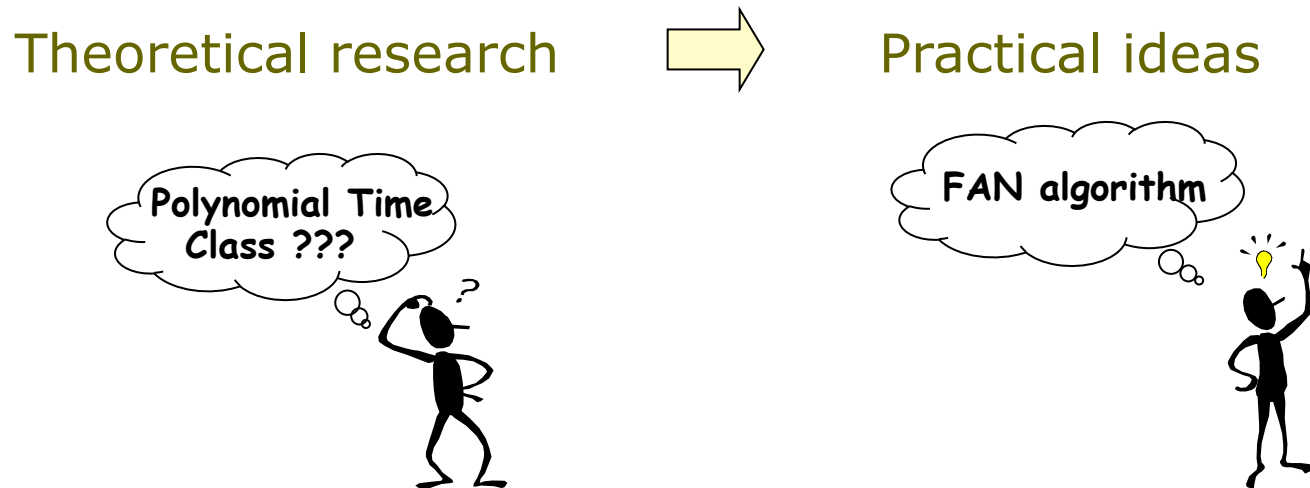
- Analysis of test generation complexity.
  - Clarify the complexity of test generation algorithms.
- Classification of sequential circuits.
  - Class of **combinational** test generation complexity.
  - Class of **acyclic** test generation complexity.

# Theory is the mother of practice

Necessity is the mother of invention

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Theoretical research / fundamental research are necessary  
for practical research.



Analysis of test generation complexity            Efficient ATPG

Classification of sequential circuits            Optimal design for testability

# Complexity of test generation

## NP-completeness

[Fujiwara, MIT Press, 1985]

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- A Boolean expression is *satisfiable* iff there exists some assignment of zeros and ones to the variables that gives the expression the value 1.
- *SAT (Satisfiability)*: Is a Boolean expression satisfiable?
- *Theorem 1* [Cook 1971]: SAT is NP-complete.
- U-SAT: Satisfiability problem for *unate* expression
- *Theorem 2* [Fujiwara 1982]: U-SAT is solvable in time  $O(L)$  where  $L$  is the length of an expression.

# Complexity of test generation

## NP-completeness

[Fujiwara, MIT Press, 1985]

- *Fault detection* (FD): Is a given single stuck-at fault detectable?
  - kM-FD: Fault detection problem for k-level monotone circuits
  - kU-FD: Fault detection problem for k-level unate circuits

- *Theorem 3* [Fujiwara 1982]:
  - 3M-FD is NP-complete.
  - Hence,
  - 3U-FD and FD are NP-complete.

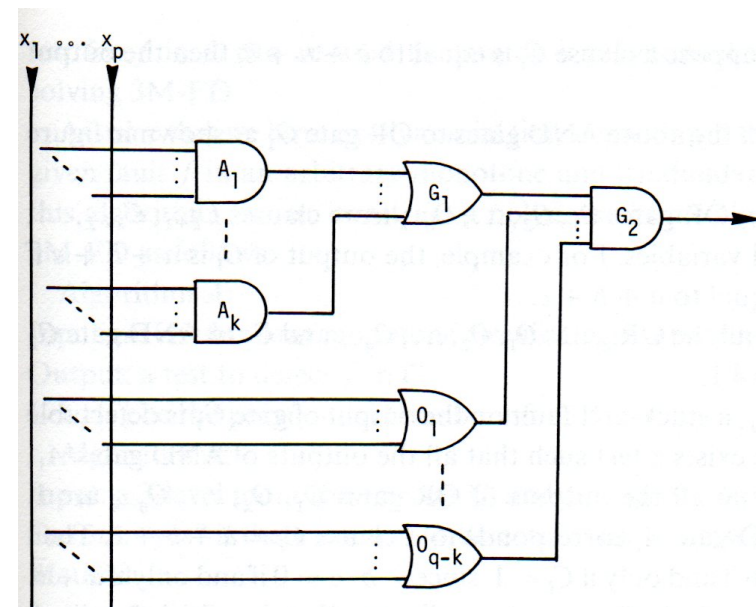


Figure 4.1  
A 3-level monotone circuit  $Q_1$



# Complexity of test generation

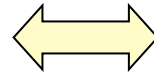
## SAT versus FD

[Fujiwara, MIT Press, 1985]

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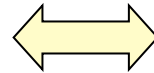
### □ Observation

SAT is NP-complete.



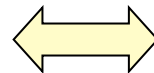
FD is NP-complete.

U-SAT is solvable in  $O(L)$ .



U-FD is NP-complete.

M-SAT is solvable in  $O(L)$ .



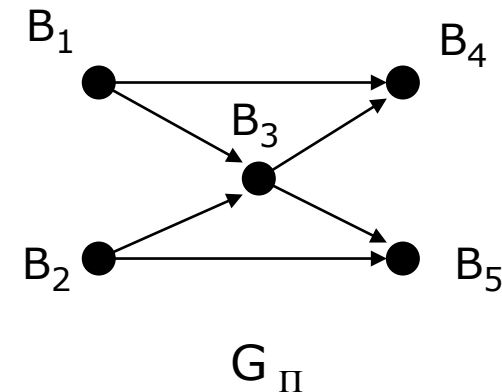
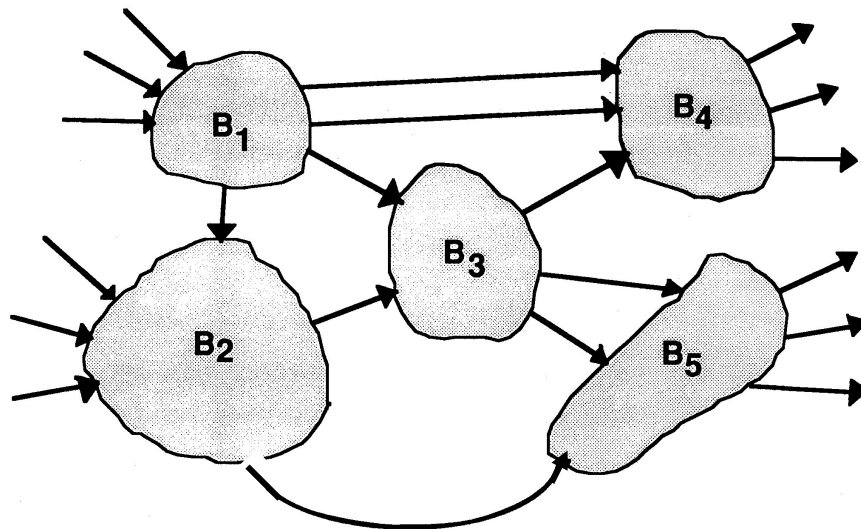
M-FD is NP-complete.

# Complexity of test generation

## Polynomial time class

[Fujiwara, MIT Press, 1985]

- A combinational circuit  $C$  is said to be *k-bounded* if there exists a partition  $\Pi = \{B_1, B_2, \dots, B_t\}$  such that
  - (1) the number of inputs of each block  $B_i$  is at most  $k$ , and
  - (2) graph  $G_\Pi$  has no cycle.



# Complexity of test generation

## $k$ -bounded circuits

[Fujiwara, MIT Press, 1985]

- **Theorem 4** [Fujiwara 1982]: Let  $C$  be a  $k$ -bounded circuit. Then there is an algorithm of time complexity  $O(16^k m)$  to find a test for a single stuck-at fault in  $C$ , where  $m$  is the number of lines in  $C$ .

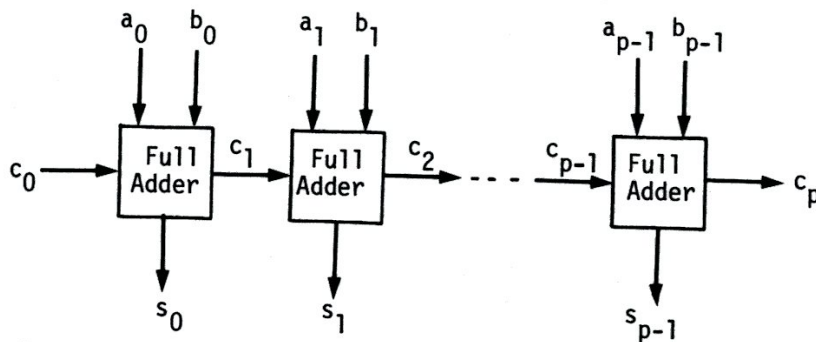


Figure 4.3  
Ripple-carry adder

$3$ -bounded circuit

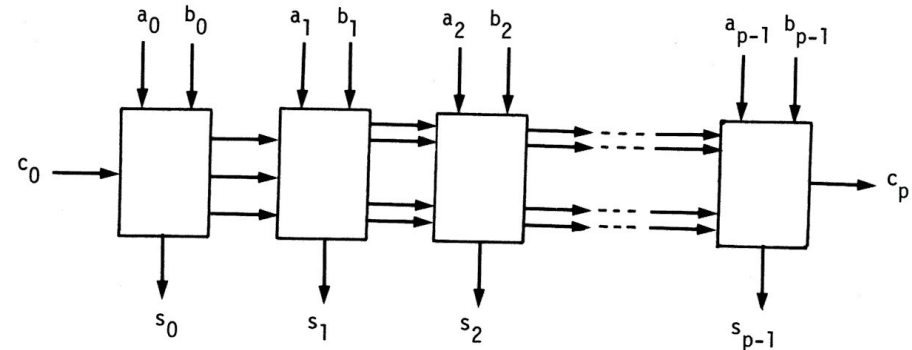


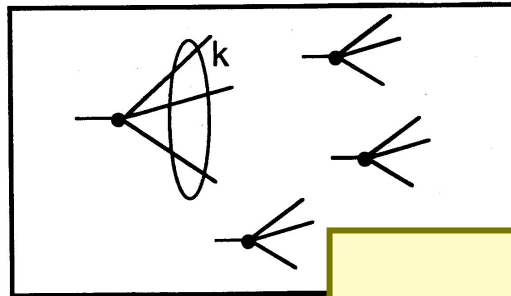
Figure 4.4  
Gate-minimum  $p$ -bit adder

$6$ -bounded circuit

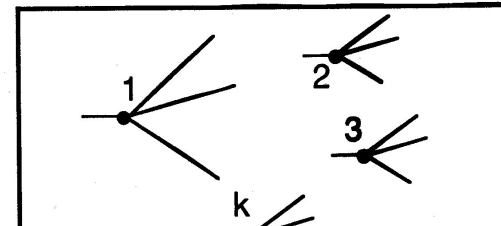
# Complexity of test generation

## k-fanout-limited vs. k-fanout-point-bounded

[Fujiwara, MIT Press, 1985]



k-fanout-li



- k-FL-FD: Fault d
  - k-FPB-FD: Fault d
- circuits

In the proof of this theorem, an algorithm is presented which requires the enumeration of at most  $4^k$  combinations of values  $(0,1,D,D')$  on fanout-points.

This algorithm is the origin of the FAN algorithm.

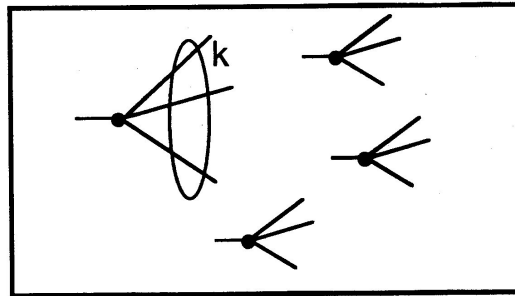
- *Theorem 5* [Fujiwara 1982]
  - k-FL-FD is NP-complete if  $k \geq 2$ .
  - k-FPB-FD is solvable in  $O(4^k m)$  where  $m$  is the number of lines in  $C$ .



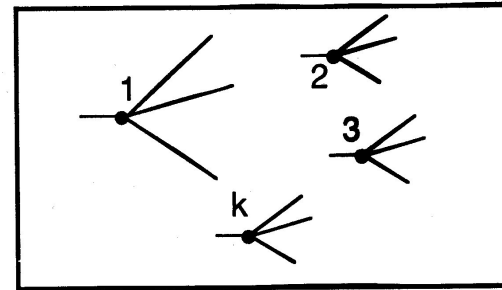
# Complexity of test generation

## k-fanout-limited vs. k-fanout-point-bounded

[Fujiwara, MIT Press, 1985]



k-fanout-limited



k-fanout-point-bounded

- **Observation:**
  - k-FL-FD is NP-complete even if  $k$  is a constant.
  - k-FPB-FD is solvable in  $O(m)$  if  $k$  is a constant.
- The complexity of test generation is affected **not** by the number of fanout branches from a fanout point **but** by the number of fanout points.

# Complexity of test generation

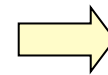
Theory is the mother of practice

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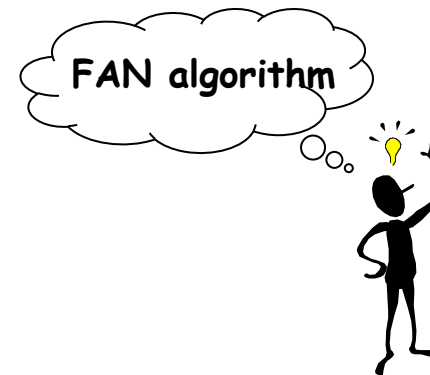
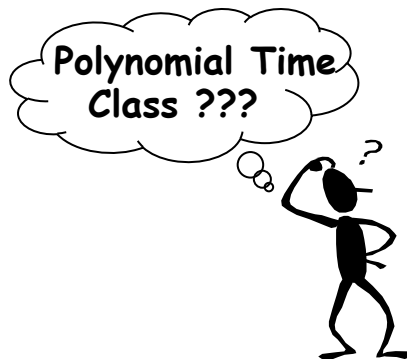
[Fujiwara, MIT Press, 1985]

The algorithm shown in the proof of the theorems is the origin of the FAN algorithm.

Analysis of test generation complexity



Efficient ATPG



# Heuristics of the FAN algorithm

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[Fujiwara, MIT Press, 1985]

- Strategy 1: In each step of the algorithm, determine as many signal values as possible that can be *uniquely implied*.

# Heuristics of the FAN algorithm

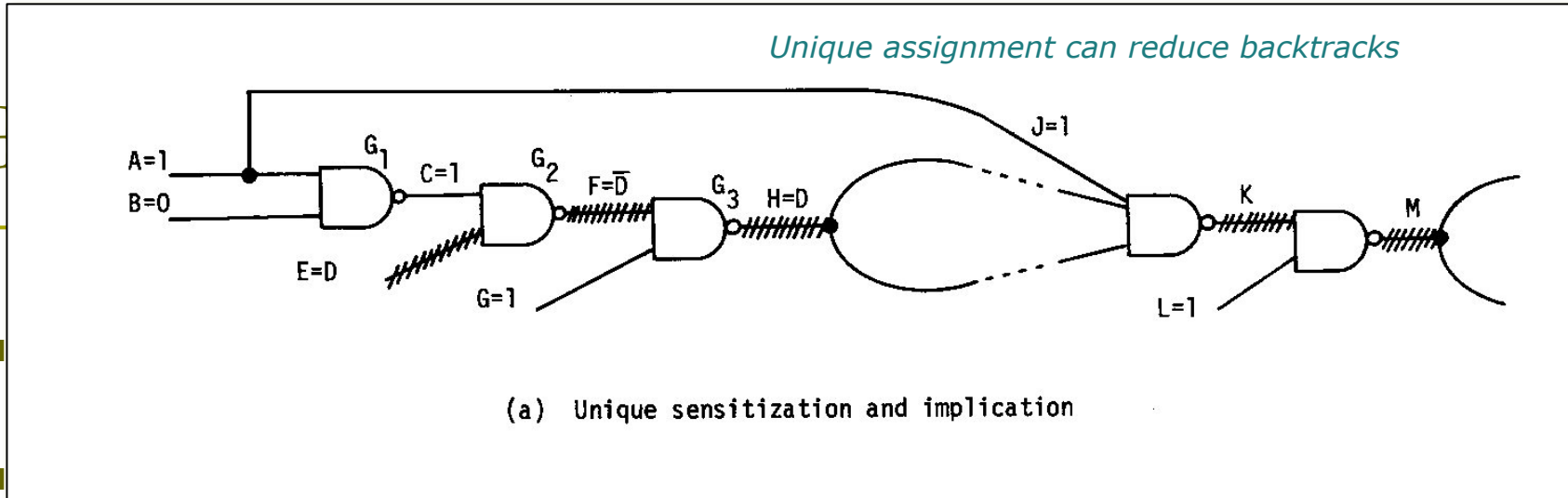
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[Fujiwara, MIT Press, 1985]

- ❑ Strategy 1: In each step of the algorithm, determine as many signal values as possible that can be *uniquely implied*.
- ❑ Strategy 2: Assign a fault signal D or D' that is *uniquely determined* or implied by the fault in question.

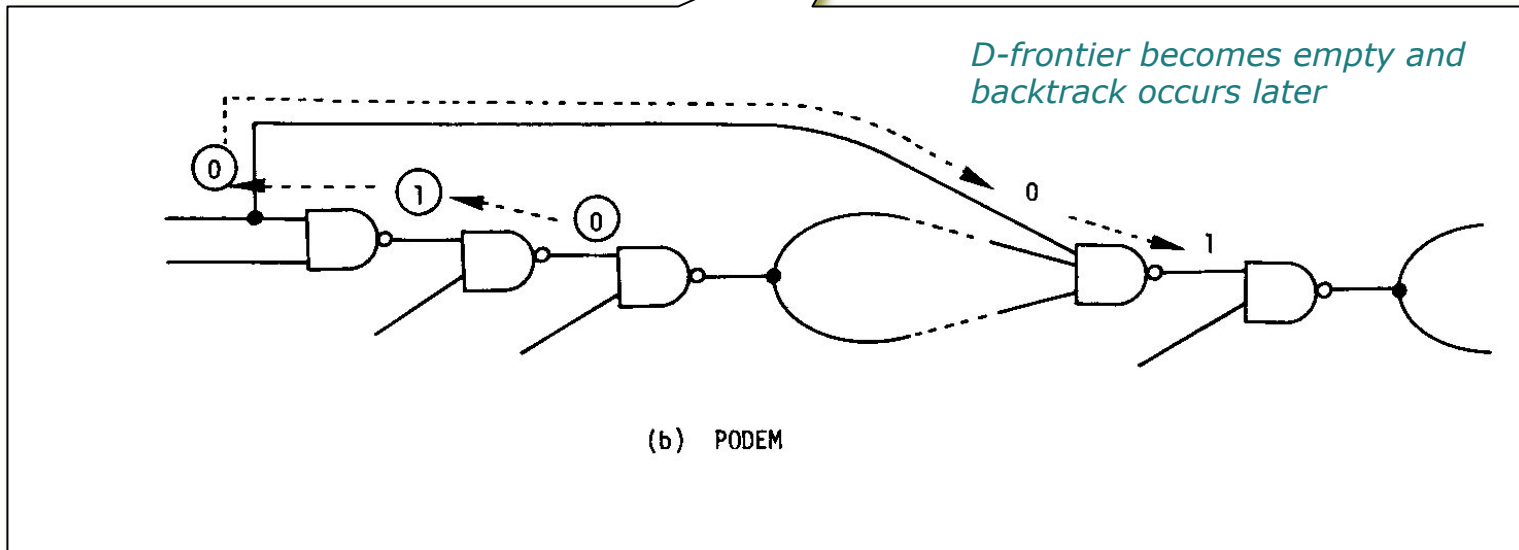


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implied by the fault in question.

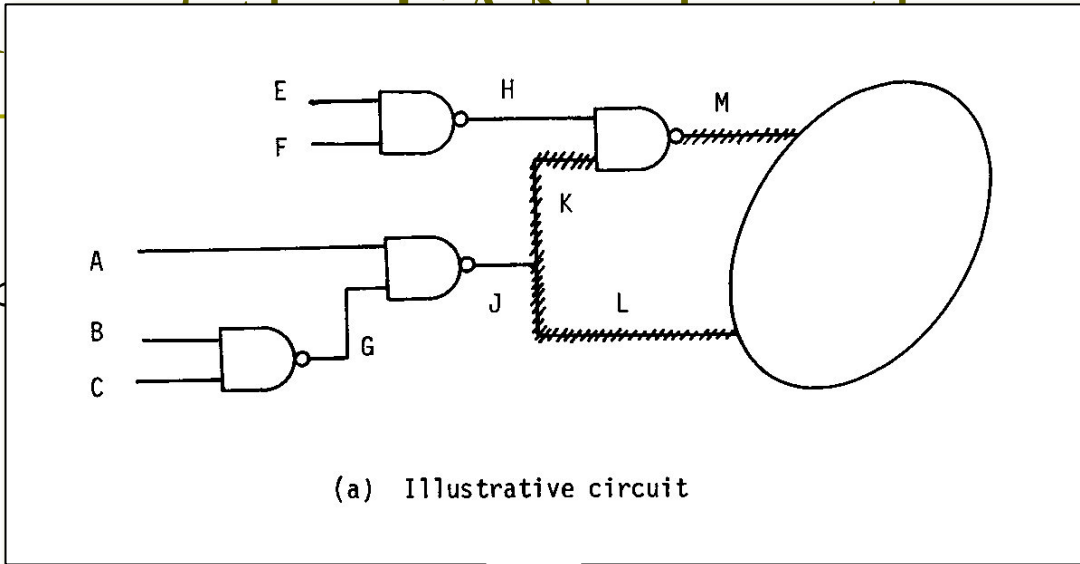
- Strategy 3: When the D-frontier consists of a single gate, apply a *unique sensitization*.



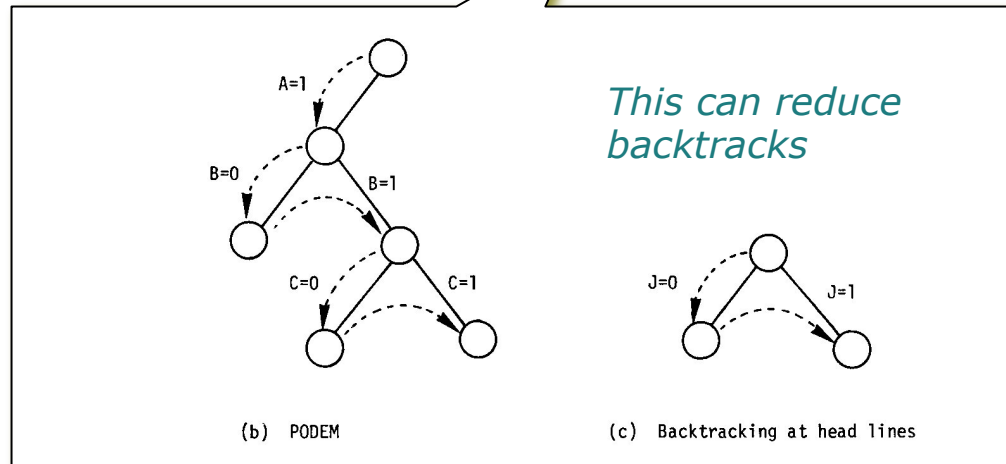
# Heuristics

## Algorithm 1

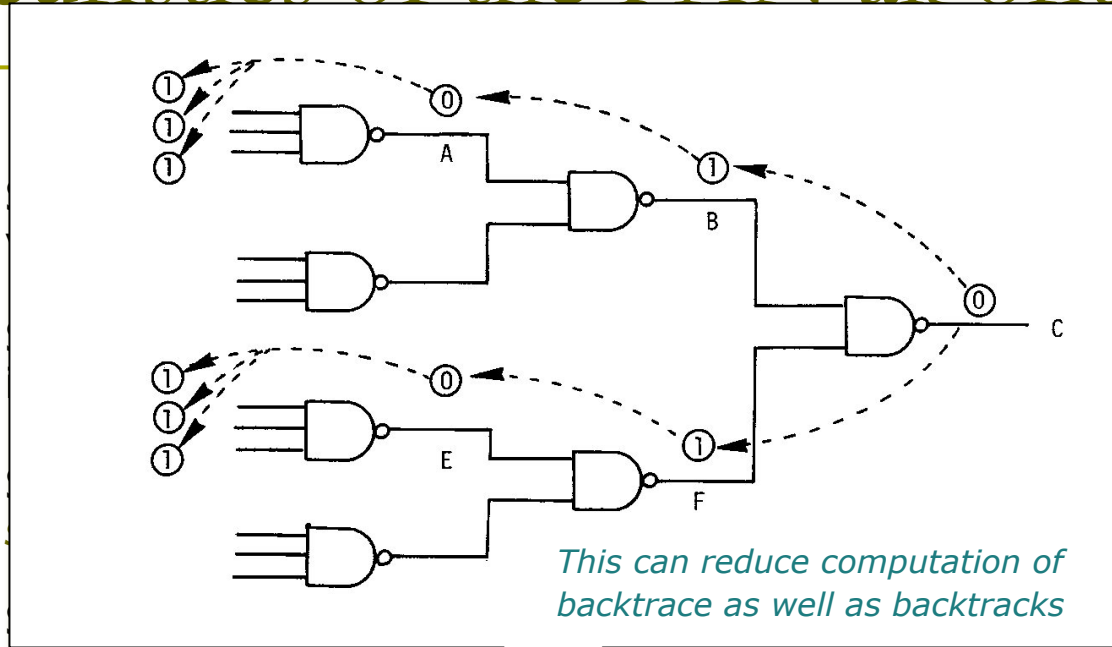
- Strategy 1: Assign values as possible
- Strategy 2: Assign values implied by
- Strategy 3: *Sensitization.*
- Strategy 4: Stop the backtrace at a *head line*, and postpone the line justification for the head line to later.



[1985]  
 by signal  
 ermined or  
 ply a *unique*



# Heuristics of the FAN algorithm



- [T Press, 1985]
- e as many signal
- *uely determined* or
- gate, apply a *unique*
- postpone the line
- justification for the head e to later.
- Strategy 5: *Multiple backtracing* (concurrent backtracing of more than one path) is more efficient than backtracing along a single path.

# Heuristics of the FAN algorithm

[Fujiwara, MIT Press, 1985]

- Strategy 1: In each step of the algorithm, determine as many signal values as possible that can be *uniquely implied*.

- Strategy 2: *PODEM* assigns a binary value only to *primary inputs*.  
So, backtracks occur at primary inputs.

- Strategy 3: *FAN* assigns a binary value only to *head lines* and *fanout points*.  
So, backtracks occur at headlines and fanout points.

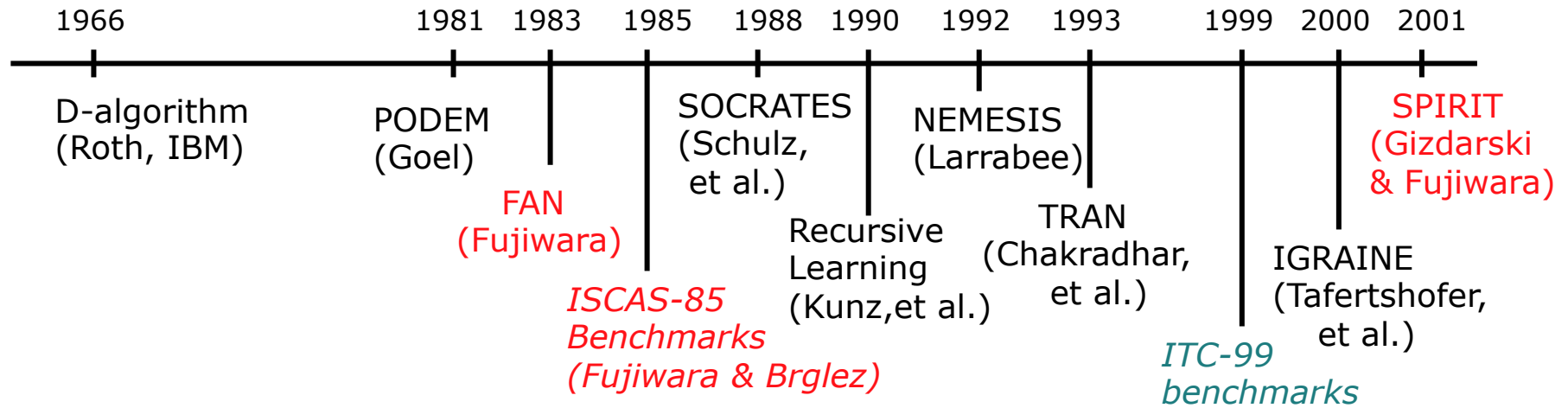
*This is effective to reduce the number of backtracks.*

- Strategy 4: ...
- Strategy 5: ...
- Strategy 6: In the multiple backtrace, if an objective at a fanout point  $p$  has a contradictory requirement, stop the backtrace so as to *assign a binary value to the fanout point*.



# History of test generation algorithms

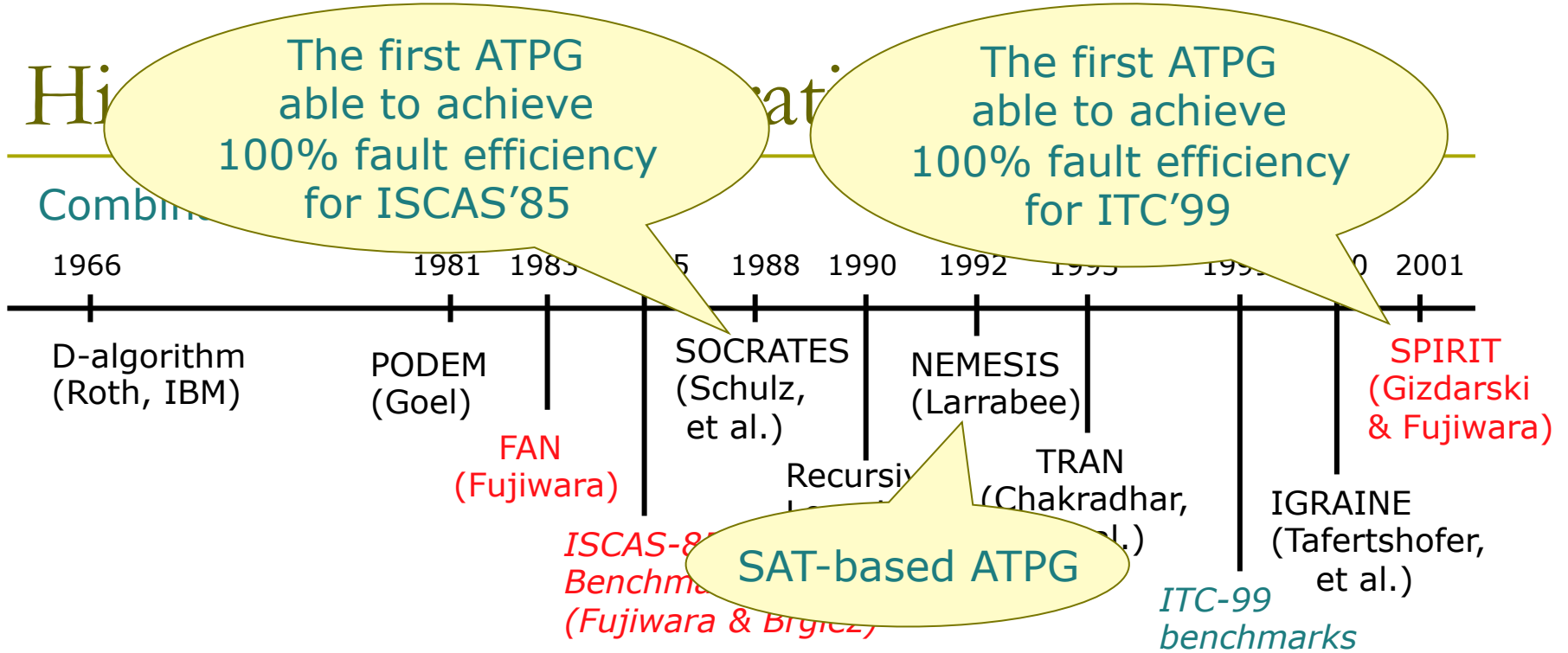
## Combinational ATPG



Hi

rat

Combined



The first ATPG able to achieve 100% fault efficiency for ISCAS'85

The first ATPG able to achieve 100% fault efficiency for ITC'99

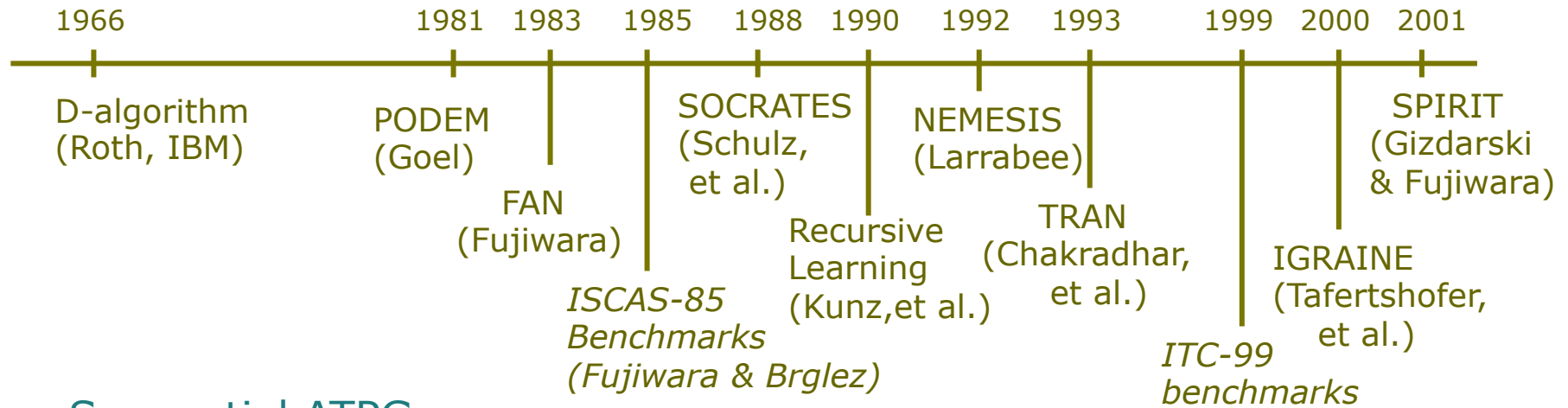
SAT-based ATPG

ISCAS-85 Benchmarks (Fujiwara & Bricez)

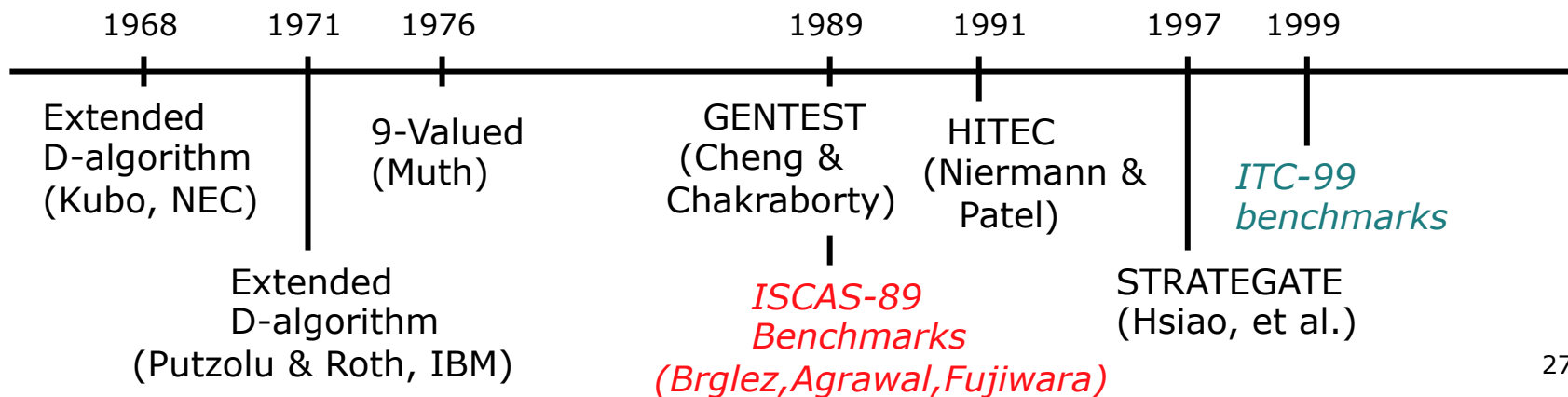
ITC-99 benchmarks

# History of test generation algorithms

## Combinational ATPG



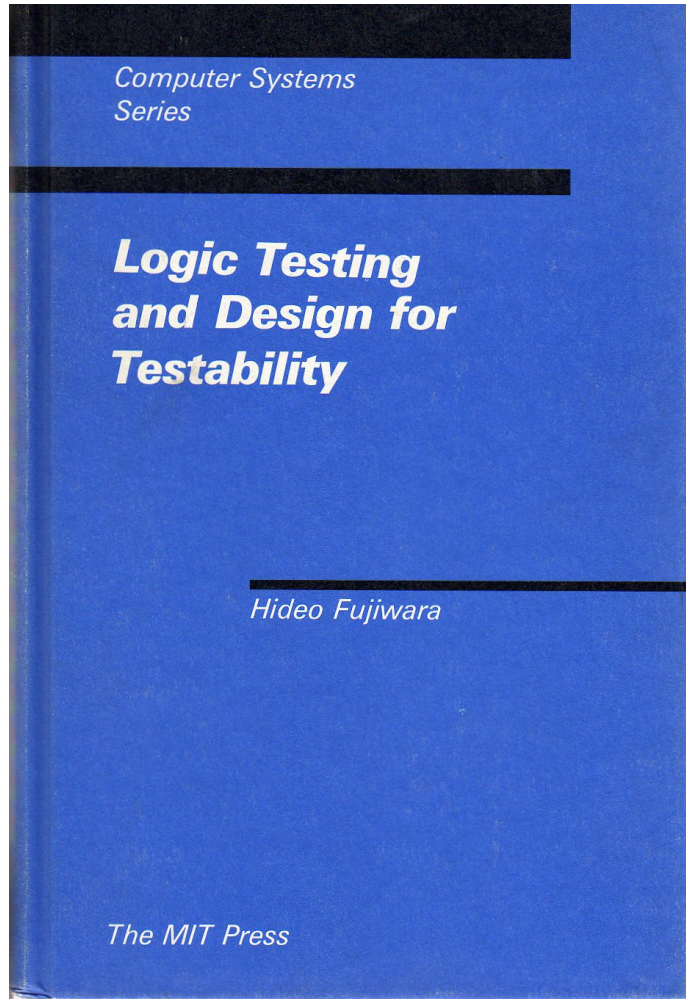
## Sequential ATPG



Hideo Fujiwara, *Logic Testing and Design for Testability*, The MIT Press, 1985

<https://mitpress.mit.edu/index.php?q=books/logic-testing-and-design-testability>

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Hardback



Paperback

Thank you

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