On the Computational Complexity of System Diagnosis

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Abstract—**In this paper we analyze the computational complexity of system diagnosis. We show that several problems for instantaneous and sequential fault diagnosis of systems are polynomially complete and that for single-loop systems these problems are solvable in polynomial time.**

Index Terms—**Fault diagnosis, polynomial time algorithm, polynomially complete, self-diagnosable systems, Turing machines.**

I. Introduction

THE AVAILABILITY of computers is a matter of prime concern, as well as their reliability, and consequently formal studies of self-diagnosability are required. A number of papers dealing with various aspects of self-diagnosable systems have appeared. Preparata *et al.* [1] first introduced a graph-theoretic model of digital systems for the purpose of diagnosis of multiple faults, and presented methods of optimal connection assignments for one-step and sequential fault-diagnosis procedures. In this model a system is made up of a number of units where each unit is assumed to be tested by some other units. With this model, necessary and sufficient conditions were given for such systems to be diagnosable for at most t faulty units [1]–[6].

Moreover, the problem of identifying the faulty units based on the test outcomes of the system was also investigated [7]–[9]. However, in general, the problem of finding a minimum set of faulty units is known to be polynomially complete [7]. That is, this problem can be solved in polynomial time if and only if the traveling salesman, knapsack problem, etc., can be solved in polynomial time. In this paper we show that several problems for instantaneous and sequential fault diagnosis of systems are polynomially complete and that for single-loop systems these problems are solvable in polynomial time.

II. Preliminaries

Consider a system of n operating units capable of testing the correctness of one another. The testing arrangements can be represented by a directed graph $G = (V, E)$, where the set of vertices $V = \{u_1, u_2, \cdots, u_n\}$ represents the units. For u_i, $u_j \in V$, there is an edge from vertex u_i to u_j; i.e., $(u_i, u_j) \in E$, if and only if u_i tests u_j. The testing unit u_i evaluates the tested unit u_j as either fault-free or faulty, the evaluation being meaningful only if u_i is fault-free. The test outcome is indicated by the weight a_{ij} on the edge $(u_i, u_j) \in E$ and obeys the following rule:

$$a_{ij} = 0, \quad \text{if } u_i \text{ and } u_j \text{ are fault-free}$$
$$a_{ij} = 1, \quad \text{if } u_i \text{ is fault-free and } u_j \text{ is faulty}$$
$$a_{ij} = 0, 1, \quad \text{if } u_i \text{ is faulty.}$$

The test outcomes $\{a_{ij}\} \equiv \sigma$ is termed the syndrome of the system. Given a directed graph $G = (V, E)$ of a system S and a syndrome $\sigma = \{a_{ij}\}$ of S, the fundamental problem is to identify the faulty units. It should be noted that each feasible set of faulty units F must satisfy the following two conditions:

1) $a_{ij} = 1$ for all $(u_i, u_j) \in E$, such that $u_i \notin F$ and $u_j \in F$;
2) $a_{ij} = 0$ for all $(u_i, u_j) \in E$, such that $u_i, u_j \in F$.

We call such subset F of V to be a *consistent fault set* of S with respect to σ.

Definition 1: A system S, represented by a directed graph $G = (V, E)$, is said to be one-step t-fault diagnosable if, for any syndrome σ, there exists at most one consistent fault set F with respect to σ such that $|F| \leq t$, where $|X|$ denotes the cardinality of a set X.

It is easy to see that in a one-step t-fault diagnosable system all faulty units can be identified, provided the number of faulty units present does not exceed t.

Let $\boldsymbol{F}_{t,\sigma}$ be the set of all consistent fault sets with respect to σ such that the number of faulty units present does not exceed t; i.e., $\boldsymbol{F}_{t,\sigma} = \{F_i \mid |F_i| \leq t, F_i \text{ is a consistent fault set with respect to } \sigma\}$.

Definition 2: A system S, represented by a directed graph G, is said to be sequentially t-fault diagnosable if, for any syndrome σ, either $\boldsymbol{F}_{t,\sigma} = \phi$ or $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i \neq \phi$.

In a sequentially t-fault diagnosable system, if the number of faulty units present does not exceed t, then $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$ is not empty if there exists at least one faulty unit. Thus we can regard all the units belonging to the intersection as faulty. Hence there exists a sequence of applications of tests and repairs of identified faulty units that allows all faulty units originally present to be identified.

Let $\boldsymbol{P}$ be the class of languages accepted by deterministic polynomial time-bounded one-tape Turing machines, and $\boldsymbol{NP}$ the class of languages accepted by nondeterministic polynomial time-bounded one-tape Turing machines (see Aho *et al.* [13]). The problem "Is $\boldsymbol{P} = \boldsymbol{NP}$?" is a longstanding open problem in complexity theory. The notion of "P-complete" used here is that of Sahni [12].

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Definition 3: A problem P_1 is said to be P-reducible to a problem P_2 (written $P_1 \propto P_2$) if the existence of a deterministic polynomial time algorithm for P_2 implies the existence of a deterministic polynomial time algorithm for P_1.

Definition 4: Two problems, P_1 and P_2, are P-equivalent if $P_1 \propto P_2$ and $P_2 \propto P_1$.

Definition 5: A problem P_i is said to be P-complete if P_i has a deterministic polynomial time algorithm if and only if $\boldsymbol{P} = \boldsymbol{NP}$. Let $\boldsymbol{PC}$ be the equivalence class of P-equivalent problems being P-complete.

III. Complexity of System Diagnosis

One-step fault diagnosis is to identify all faulty units in a system instantaneously. In a one-step t-fault diagnosable system, there exists a unique consistent fault set, provided that the number of faulty units does not exceed t. However, in general, there exists more than one consistent fault set. Hence, we regard the minimum consistent fault set as the most likely consistent fault set. The problem of finding such a minimum fault set is known to be P-complete [7].

Sequential fault diagnosis is a sequence of applications of tests and repairs of identified faulty units that allows all faulty units originally present to be identified. Hence, in sequential fault-diagnosis procedures it is important to compute $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$.

Problem 1 (P1): Given a system S, represented by a directed graph $G = (V, E)$, a syndrome σ, and a positive integer t, does there exist a consistent fault set F of S with respect to σ such that $|F| \le t$?

Problem 2 (P2): Given a system S, represented by a directed graph $G = (V, E)$, a syndrome σ, a unit u, and a positive integer t, does there exist F_i such that $u \notin F_i$ and $F_i \in \boldsymbol{F}_{t,\sigma}$?

Problem 3 (P3): Given a system S, represented by a directed graph $G = (V, E)$, a syndrome σ, a unit u, and a positive integer t, does the unit u belong to $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$?

Problem 4 (P4): Given a system S, represented by a directed graph G, a syndrome σ, and a positive integer t, find $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$.

Problem 5 (P5): Given a system S, represented by a directed graph G, a syndrome σ, and a positive integer t, find at least one unit in $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$ if not empty.

Problem 6 (P6): Given a system S, represented by a directed graph $G = (V, E)$, a syndrome σ, and a unit u, does there exist F_i such that $u \notin F_i$ and $F_i \in \boldsymbol{F}_{|V|,\sigma}$?

Problem 7 (P7): Given a system S, represented by a directed graph $G = (V, E)$, and a syndrome σ, find $\bigcap_{F_i \in \boldsymbol{F}_{|V|,\sigma}} F_i$.

In this section, we show that the previously stated problems P1–P5 for sequential and one-step fault diagnosis are P-complete and that problems P6 and P7 are solvable in polynomial time.

Lemma 1

P1 $\propto$ P2.

Proof: Given a system S represented by a directed graph $G = (V, E)$, a syndrome $\sigma = \{a_{ij} | (u_i, u_j) E\}$, and a positive integer t, construct a system S' represented by a directed graph $G' = (V', E')$ such that $V' = V \cup \{u_p, u_q\}$ and $E' = E \cup \{(u_p, u_q), (u_q, u_p)\}$. Let σ' be a syndrome of S' such that $\sigma' = \sigma \cup \{a_{pq} = 0, a_{qp} = 0\}$.

It can easily be shown that a set F is a consistent fault set of S with respect to σ if and only if F is a consistent fault set of S' with respect to σ' such that $u_p, u_q \notin F$. Hence we can see that a polynomial time algorithm for P2 implies a polynomial time algorithm for P1. Q.E.D.

Maheshwari and Hakimi [7] have shown that the problem of finding the minimum consistent fault set is P-complete, and thus P1 $\in \boldsymbol{PC}$. It can easily be shown that $\boldsymbol{P} = \boldsymbol{NP}$ implies P2 $\in \boldsymbol{P}$. From this and Lemma 1, we have P2 $\in \boldsymbol{PC}$. It can also be shown that Problems P2, P3, and P4 are P-equivalent. Hence, we have the following theorem.

Theorem 1

P1, P2, P3, and P4 $\in \boldsymbol{PC}$.

To diagnose a given system sequentially, it is not always necessary to find all the units in $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$, but it is necessary to find at least one unit in $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$. This is Problem 5, but even for P5 we can show it to be P-complete in the following theorem.

Given a system S, represented by a directed graph $G = (V, E)$, a syndrome σ, and a positive integer t, let us define the set X_u corresponding to a unit u in $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i$ as follows. X_u is the smallest set of vertices such that 1) $u \in X_u$, and 2) if $u_j \in X_u$, $(u_i, u_j) \in E$ and $a_{ij} = 0$, then $u_i \in X_u$. Let S' be the system represented by the directed graph $G' = (V', E')$ such that $V' = V - X_u$ and $E' = \{(u_i, u_j) | (u_i, u_j) \in E, u_i, u_j \in V'\}$. Let $\sigma' = \{a_{ij} | a_{ij} \in \sigma, \ (u_i, \ u_j) \in E'\}$, $t' = t - |X_u|$ and $\boldsymbol{F}_{t',\sigma'} = \{F_i' \,|\, |F_i'| \le t', F_i'$ is a consistent fault set of S' with respect to $\sigma'\}$. For the systems S and S', we have the following lemma.

Lemma 2

1) For any $F_i \in \boldsymbol{F}_{t,\sigma}$, $X_u \subseteq F_i$.
2) $\boldsymbol{F}_{t',\sigma'} = \{F_i' | F_i' = F_i - X_u, F_i \in \boldsymbol{F}_{t,\sigma}\}$.
3) $\bigcap_{F_i \in \boldsymbol{F}_{t,\sigma}} F_i = \bigcap_{F_{i'} \in \boldsymbol{F}_{t',\sigma'}} F_i' \cup X_u$.

Proof: 1) Suppose that there exists a set F such that $X_u \nsubseteq F$ and $F \in \boldsymbol{F}_{t,\sigma}$. From this and the definition of X_u we can see that there exists an edge $(u_i, u_j) \in E$ such that u_i, $u_j \in X_u$, $u_i \notin F$, $u_j \in F$, and $a_{ij} = 0$. This contradicts that F is a consistent fault set of S with respect to σ.

2) For any $F_i \in \boldsymbol{F}_{t,\sigma}$, $X_u \subseteq F_i$, and thus $F_i - X_u \in \boldsymbol{F}_{t',\sigma'}$. Hence

$$\boldsymbol{F}_{t',\sigma'} \supseteq \{F_i' | F_i' = F_i - X_u, F_i \in \boldsymbol{F}_{t,\sigma}\}.$$

Conversely, we show that for any $F_i' \in \boldsymbol{F}_{t',\sigma'}$ there exists a set F_i such that $F_i' = F_i - X_u$ and $F_i \in \boldsymbol{F}_{t,\sigma}$.

For any $F_i' \in \boldsymbol{F}_{t',\sigma'}$, F_i' is a consistent fault set of S' with respect to σ'. Hence, $a_{ij} = 1$ for all $(u_i, u_j) \in E$ such that $u_i \notin F_i' \cup X_u$ and $u_j \in F_i'$, and $a_{ij} = 0$ for all $(u_i, u_j) \in E$ such that u_i, $u_j \notin F_i' \cup X_u$. From the definition of X_u, we have that $a_{ij} = 1$ for all $(u_i, u_j) \in E$ such that $u_i \notin F_i' \cup X_u$ and $u_j \in X_u$.

Therefore, $F_i' \cup X_u$ is a consistent fault set of S with respect to σ. Clearly, $F_i' \cup X_u = \phi$ and $|F_i' \cup X_u| =$

$|F_i'| + |X_u| \le t$. Hence $F_i = F_i' \cup X_u$ is in $F_{t,\sigma}$, and

$$F_{t',\sigma'} \subseteq \{F_i' \mid F_i' = F_i - X_u, F_i \in F_{t,\sigma}\}.$$

3) From 1) and 2) it is clear that

$$\bigcap_{F_i' \in F_{t',\sigma'}} F_i' = \bigcap_{F_i \in F_{t,\sigma}} (F_i - X_u) = \bigcap_{F_i \in F_{t,\sigma}} F_i - X_u.$$

Hence

$$\bigcap_{F_i \in F_{t,\sigma}} F_i = \bigcap_{F_i' \in F_{t',\sigma'}} F_i' \cup X_u. \qquad \text{Q.E.D.}$$

Theorem 2

P5 $\in$ ***PC***.

Proof: From Theorem 1, P4 $\in$ ***PC***. Clearly, P5 $\propto$ P4. Hence it suffices to show P4 $\propto$ P5.

Given a system S, represented by a directed graph $G = (\bar{v}, E)$, a syndrome σ, and a positive integer t, then we can find $\bigcap_{F_i \in F_{t,\sigma}} F_i$, using the following algorithm.

Algorithm for P4

<u>begin</u>
  OUTPUT $\leftarrow \phi$ (the empty set);
  <u>while</u> $t > 0$ do
    <u>begin</u>
      use the algorithm for P5 to find at least one unit u in $\bigcap_{F_i \in F_{t,\sigma}} F_i$ if not empty;
      <u>if</u> the algorithm judges the intersection to be empty
        <u>then</u> stop
        <u>else</u>
          <u>begin</u>
            construct the set X_u corresponding to u;
            delete all units in X_u from the system;
            $t \leftarrow t - |X_u|$;
            OUTPUT $\leftarrow$ OUTPUT $\cup X_u$
          <u>end</u>
    <u>end</u>
<u>end</u>

From Lemma 2, clearly this algorithm terminates and then OUTPUT $= \bigcap_{F_i \in F_{t,\sigma}} F_i$. If the algorithm for P5 can be done in polynomial time, then the previous algorithm for P4 is a polynomial time algorithm. Hence P4 $\propto$ P5. Q.E.D.

Theorem 3

P6 and P7 are solvable in polynomial time.

Proof: Let $x_1, x_2, \cdots, x_n$ be the binary variables corresponding to the units $u_i, u_2, \cdots, u_n$, respectively, such that $x_i = 1$ if unit u_1 is fault-free, and $x_i = 0$ if unit u_i is faulty. From Preparata [2], we see that a set F is a consistent fault set of a system S with respect to a syndrome σ if and only if the assignment of 0's to the variables in F and of 1's to the variables not in F gives the following Boolean expression the value 1:

$$\prod_{a_{ij} \in \sigma} (\bar{x}_i \vee x_j \bar{a}_{ij} \vee \bar{x}_j a_{ij}).$$

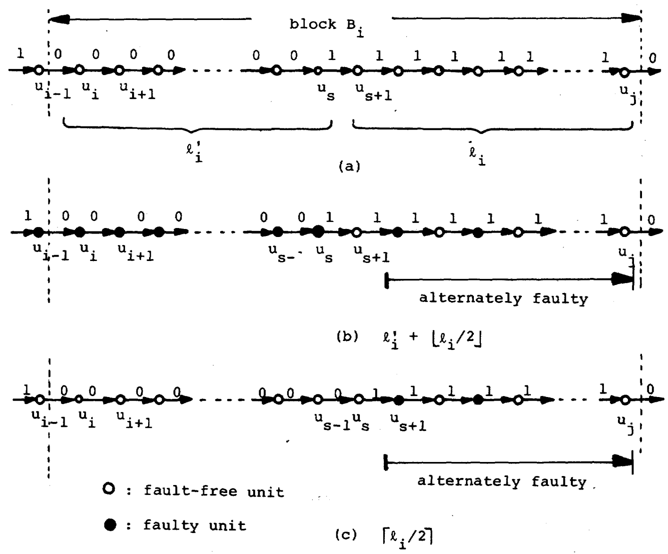

Fig. 1. Illustration for Lemma 3.

Similarly, for a consistent fault set F, which does not contain a given unit u_k, the previous expression can be reduced to the following expression:

$$\prod_{\substack{a_{ij} \in \sigma \\ i,j \ne k}} (\bar{x}_i \vee x_j \bar{a}_{ij} \vee \bar{x}_j a_{ij}) \cdot \prod_{a_{kj} \in \sigma} (x_j \bar{a}_{kj} \vee \bar{x}_j a_{ij}) \cdot \prod_{a_{ik} \in \sigma} (\bar{x}_i \vee \bar{a}_{ik}).$$

Therefore, the just given Boolean expression is satisfiable [14] if and only if $u_i \notin F$ for some $F \in F_{n,\sigma}$. This expression is in 2-conjunctive normal form, and thus P6 is P-reducible to the 2-satisfiability problem which has a polynomial time algorithm [14]. Hence P6 is solvable in polynomial time.

We can see that $u \notin F$ for some $F \in F_{n,\sigma}$ if and only if $u \notin \bigcap_{F \in F_{n,\sigma}} F$. Therefore, using a polynomial time algorithm for P6, we can determine whether $u \in \bigcap_{F \in F_{n,\sigma}} F$ for each $u \in V$, and thus we can construct $\bigcap_{F \in F_{n,\sigma}} F$ in polynomial time. Hence P7 is solvable in polynomial time. Q.E.D.

IV. Single-Loop Systems

A single-loop system is a system consisting of a cycle of units $u_1, u_2, \cdots, u_n$ in which unit u_i tests unit u_{i+1}, $1 \le i \le n-1$ and unit u_n tests unit u_1. The necessary and sufficient condition is given for such single-loop systems to be sequentially t-fault diagnosable [1], [2]. In Section III we have shown that, in general, Problems P1–P5 are P-complete. In this section, we show that for single-loop systems Problems P1–P5 are all solvable in polynomial time.

Given a single-loop system S, represented by a directed graph $G = (V, E)$ and a syndrome σ, let us partition the loop into blocks of units where each block $B_i = \{u_i, u_{i+1}, \cdots, u_s, \cdots, u_j\}$ has the weight pattern of the form $0 \ldots 01 \ldots 1$, as illustrated in Fig. 1(a). In the case when the test outcomes are all 1's, this partition is *one* partition having only one block V.

Let (u_i, u_j) be an edge in E; u_i is called the tail and u_j the head of the edge (u_i, u_j); $\lfloor a \rfloor$ denotes the greatest integer not exceeding a, while $\lceil a \rceil$ denotes the smallest integer not smaller than a.

Lemma 3

Let $B_1, B_2, \cdots, B_k$ be all blocks of single-loop system S with respect to syndrome σ. For a minimum consistent fault set $\hat{F}$ with respect to σ, we have

$$|\hat{F}| = \sum_{i=1}^{k} \lceil l_i/2 \rceil$$

where l_i is the number of edges of weight 1 whose tail is in block B_i.

Proof: If the syndrome σ is σ_1 where all test outcomes are 1's, then we have $k = 1$; i.e., block B_1 contains all units in the system. In this case, clearly, the cardinality of the minimum consistent fault set is $\lceil l_1/2 \rceil$.

If σ is a syndrome where at least one test outcome 0 appears, then we can let l_i' be the number of edges of weight 0 whose heads are in block B_i, where $B_i = \{u_i, u_{i+1}, \cdots, u_s, \cdots, u_j\}$. Let u_s be the last unit which is the head of an edge of weight 0 [see Fig. 1(a)]. If u_s is faulty, then $u_i, u_{i+1}, \cdots, u_s$ are all faulty and we obtain the smallest set F_{i0} of faulty units in block B_i, as shown in Fig. 1(b). The number of such faulty units is $l_i' + \lfloor l_i/2 \rfloor$. If u_s is fault-free, then we have the smallest set F_{i1} of faulty units in B_i, as shown in Fig. 1(c). The number of such faulty units is $\lceil l_i/2 \rceil$.

Since $l_i' > 0$, we have $l_i' + \lfloor l_i/2 \rfloor \geq \lceil l_i/2 \rceil$ for all i. Therefore, F_{i1} is smaller than F_{i0}, which implies that F_{i1} is the smallest set of faulty units in B_i. From this, we can see that the union of F_{i1} for all B_i $(1 \leq i \leq k)$ is the minimum consistent fault set, the cardinality of which is $\sum_{i=1}^{k} \lceil l_i/2 \rceil$. Q.E.D.

Given a single-loop system S represented by a directed graph $G = (V, E)$ and a syndrome σ, we can compute the value l_i for each block B_i in $O(|V|)$ steps. Therefore, from Lemma 3 we have an $O(|V|)$ algorithm for computing the cardinality of the minimum consistent fault set $\hat{F}$. Clearly, $|\hat{F}| \leq t$ if and only if there exists a consistent fault set F such that $|F| \leq t$. Hence by computing $|\hat{F}|$ we can solve Problem 1, and thus we have the following theorem.

Theorem 4

For single-loop systems there exists a deterministic algorithm for P1 of time complexity $O(|V|)$, where $|V|$ is the number of units.

For single-loop systems, Problems 2 and 3 are also solvable in time complexity $O(|V|)$.

Theorem 5

For single-loop systems, there exists a deterministic algorithm for P2 and P3 of time complexity $O(|V|)$.

Proof: Let $G = (V, E)$ be the directed graph representing a single-loop system S and let σ be a syndrome. Let $\hat{F}$ be a minimum consistent fault set with respect to σ such that $u \notin \hat{F}$ for a given unit u. Clearly, $|\hat{F}| \leq t$ if and only if there exists a consistent fault set F such that $u \notin F$ and $|F| \leq t$. Therefore, there exists an $O(|V|)$ algorithm for P2 and P3 if we have an $O(|V|)$ algorithm for computing the cardinality of a minimum consistent fault set $\hat{F}$ with $u \notin \hat{F}$. Hence, to complete the proof it suffices to show that there exists an $O(|V|)$ algorithm to compute $|\hat{F}|$ with $u \notin \hat{F}$.

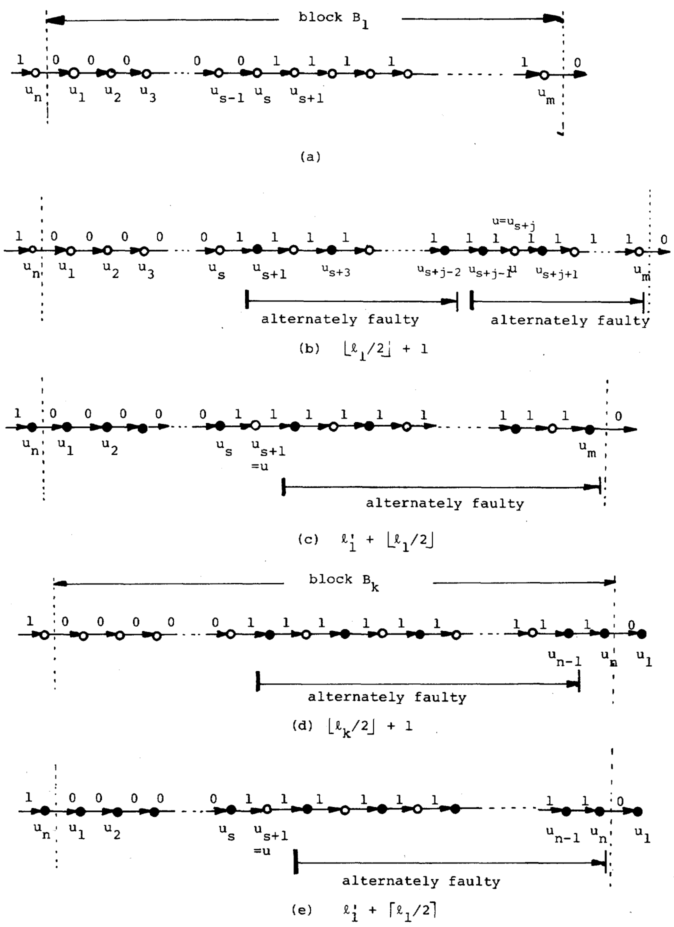

Fig. 2. Illustration for Theorem 5.

Let $V = \{u_1, u_2, \cdots, u_n\}$ and let $B_1, B_2, \cdots, B_k$ be all blocks of V partitioned according to the syndrome σ. If σ is σ_1 where all test outcomes are 1's, then we have $k = 1$; i.e., block B_1 equals V. In this case, clearly, there exists a minimum consistent fault set $\hat{F}$ with $u \notin \hat{F}$, and the cardinality of F is $\lceil n/2 \rceil$.

If σ is a syndrome where at least one test outcome 0 appears, then without loss of generality we assume $u \in B_1$, $B_1 = \{u_1, u_2, \cdots, u_s, \cdots, u_m\}$, $a_{n1} = a_{12} = \cdots = a_{s-1,s} = 0$, and $a_{s,s+1} = \cdots = a_{m-1,m} = 1$ [see Fig. 2(a)]. Let X be the set of units $u_{s+1}, u_{s+3}, u_{s+5}, \cdots$, selected alternately from u_{s+1} until u_m. Clearly, $|X| = \lceil l_1/2 \rceil$.

Case 1: $u \notin X$. Clearly, there exists a minimum consistent fault set $\hat{F}$ with $u \notin \hat{F}$, and thus $|\hat{F}| = \sum_{i=1}^{k} \lceil l_i/2 \rceil$.

Case 2: $u \in X$ and $u \neq u_{s+1}$. Suppose that $u = u_{s+j}$ and unit u is fault-free, then we have the smallest set of faulty units $\{u_{s+1}, u_{s+3}, \cdots, u_{s+j-2}; u_{s+j-1}, u_{s+j+1}, u_{s+j+3}, \cdots\}$ in block B_1, as shown in Fig. 2(b). The number of such faulty units is $\lfloor l_1/2 \rfloor + 1$. For other blocks B_i, the smallest number of faulty units is $\lceil l_i/2 \rceil$. Therefore,

$$|\hat{F}| = \lfloor l_1/2 \rfloor + 1 + \sum_{i=2}^{k} \lceil l_i/2 \rceil.$$

Case 3: $u = u_{s+1}$ and $k \geq 2$. Suppose that unit u is fault-free, then we have the smallest set of faulty units in block B_1 as shown in Fig. 2(c). The number of such faulty units is $l'_1 + \lfloor l_1/2 \rfloor$. In this case, unit u_n in block B_k is also recognized to be faulty. Therefore, we have the smallest set of faulty units in block B_k, as shown in Fig. 2(d). The number of such faulty units is $\lfloor l_k/2 \rfloor + 1$. Hence, we have

$$|\hat{F}| = \lfloor l_1/2 \rfloor + l'_1 + \lfloor l_k/2 \rfloor + 1 + \sum_{i=2}^{k-1} \lceil l_i/2 \rceil.$$

Case 4: $u = u_{s+1}$, $k = 1$, and $l_1 \geq 2$. Suppose that u is fault-free, then we have the smallest set of faulty units as shown in Fig. 2(e). The number of such faulty units is $l'_1 + \lceil l_1/2 \rceil$. Therefore,

$$|\hat{F}| = l'_1 + \lceil l_1/2 \rceil.$$

Case 5: $u = u_{s+1}$, $k = 1$, and $l_1 = 1$. Clearly, $u = u_{s+1} = u_n$, and in this case there exists no consistent fault set $\hat{F}$ with $u \notin \hat{F}$.

In any of the cases mentioned above, we can compute the cardinality of a minimum consistent fault set F with $u \notin F$ in computational time of $O(|V|)$. Q.E.D.

By applying an algorithm for P2 to each unit $u_i \in V$, we can solve Problems 4 and 5. Therefore, from Theorem 5 we can see that P4 and P5 are solved by $O(|V|)$ algorithm for single-loop systems.

Theorem 6

For single-loop systems there exists a deterministic algorithm for P4 and P5 of time complexity $O(|V|)$.

V. Conclusion

In this paper we have clarified the computational complexity of fault diagnosis in self-diagnosable systems. In general, several problems for one-step and sequentially fault diagnosis are *P*-complete. However, for single-loop systems such problems are all solvable in polynomial time.

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References

[1] F. P. Preparata, G. Metze, and R. T. Chien, "On the connection assignment problem of diagnosable systems," *IEEE Trans. Electron. Comput.*, vol. EC-16, pp. 848–854, Dec. 1967.

[2] F. P. Preparata, "Some results on sequentially diagnosable systems," in *Proc. Hawaii Int. Conf. System Sciences*, pp. 623–626, 1968.

[3] S. L. Hakimi and A. T. Amin, "Characterization of connection assignment of diagnosable systems," *IEEE Trans. Comput.*, vol. C-23, pp. 86–88, Jan. 1974.

[4] F. J. Allan, T. Kameda, and S. Toida, "An approach to the diagnosability analysis of a system," *IEEE Trans. Comput.*, vol. C-24, pp. 1040–1042, Oct. 1975.

[5] F. Barsi, F. Grandoni, and P. Maestrini, "A theory of diagnosability of digital systems," *IEEE Trans. Comput.*, vol. C-25, pp. 585–593, June 1976.

[6] S. Toida, "System diagnosis and redundant tests," *IEEE Trans. Comput.*, vol. C-25, pp. 1167–1170, Nov. 1976.

[7] S. N. Maheshwari and S. L. Hakimi, "On models for diagnosable systems and probabilistic fault diagnosis," *IEEE Trans. Comput.*, vol. C-25, pp. 228–236, Mar. 1976.

[8] T. Kameda, S. Toida, and F. J. Allan, "A diagnosing algorithm for networks," *Inform. Contr.*, vol. 29, pp. 141–148, 1975.

[9] A. M. Corluhan and S. L. Hakimi, "On an algorithm for identifying faults in a *t*-diagnosable system," in *Proc. Johns Hopkins Conf. Information Sciences and Systems*, pp. 370–378, Apr. 1976.

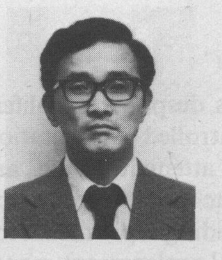
[10] S. A. Cook, "The complexity of theorem proving procedures," in *Conf. Rec. 3rd ACM Symp. Theory of Computing*, pp. 151–158, 1971.

[11] R. M. Karp, "Reducibility among combinational problems," in *Complexity of Computer Computations*, R. E. Miller and J. W. Thatcher, Ed. New York: Plenum, 1972, pp. 85–104.

[12] S. Sahni, "Computationally related problems," *SIAM J. Comput.*, vol. 5, pp. 262–279, 1974.

[13] A. V. Aho, J. E. Hopcroft, and J. D. Ullman, *The Design and Analysis of Computer Algorithms*. Reading, MA: Addison-Wesley, 1974.

[14] O. H. Ibarra and S. K. Sahni, "Polynomially complete fault detection problems," *IEEE Trans. Comput.*, vol. C-24, pp. 242–249, Mar. 1975.



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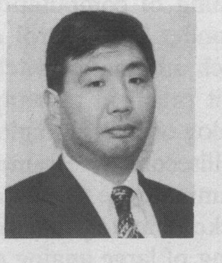

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