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Minimization of Nondeterministic Finite Local Automata

By

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1. Introduction

Gabrielián [1] has introduced a kind of automata that have nonsequential inputs in the form of arbitrary labeled directed graphs, called "local automata". Local automata are defined as labeled systems that interact and accept or reject each other. This is an interesting generalization because these automata may be interpreted as machines which accept labeled directed graphs as input.

In this paper we consider minimization problem for finding a minimum state nondeterministic finite local automaton (NDLA) equivalent to a given local automaton. We also define a class of local automata whose characteristic transition functions are deterministic, and then show that there is a reduced characteristically deterministic finite local automaton (CDLA) which uniquely exists for any CDLA.

Given a nondeterministic finite automaton (NFA), we can find a deterministic finite automaton (DFA) which is equivalent to it, e.g., using the subset construction.

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Local automata are defined as labeled systems that interact and accept or reject each other. The input to a local automaton is a labeled directed graph and thus local automata are generalizations of ordinary automata. The aim of this paper is to show a procedure for finding a minimum state nondeterministic finite local automaton equivalent to a given finite local automaton.

1. Introduction

Gabrielian [1] has introduced a kind of automata that have nonsequential inputs in the form of arbitrary labeled directed graphs, called "local automata". Local automata are defined as labeled systems that interact and accept or reject each other. This is an interesting generalization because these automata may be interpreted as machines which accept labeled directed graphs as input.

In this paper we consider minimization problem for finding a minimum state *nondeterministic finite local automaton* (NDLA) equivalent to a given local automaton. We also define a class of local automata whose characteristic transition functions are deterministic, and then show that there is a reduced *characteristically deterministic finite local automaton* (CDLA) which uniquely exists for any CDLA.

Given a *nondeterministic finite automaton* (NDA), we can find a *deterministic finite automaton* (DA) which is equivalent to it, e.g., using the subset construction. Similarly, we can find a CDLA equivalent to a given NDLA using the modified subset construction. Once an equivalent CDLA is constructed, we can obtain a reduced CDLA equivalent to the given NDLA. Therefore with an NDLA we can uniquely associate a reduced CDLA equivalent to it. Thus our problem can be reduced to finding a minimum state NDLA equivalent to a given reduced CDLA.

2. Preliminary Definitions

We first give some definitions in the following.

Definition 1. A nondeterministic (finite) local automaton (NDLA) A is a system

$$A = (\Sigma, S, M, S_0, F, \gamma, K)$$

where Σ is the input alphabet, S is the finite nonempty set of states, $M: S \times \Sigma \rightarrow 2^S$ is the transition function, $S_0 (\neq \emptyset) \subseteq S$ is the set of initial states, $F \subseteq S$ is the set of final states, $\gamma: S \rightarrow K$ is the label function, and K is the finite set of labels.

Definition 2. An "interaction" between two NDLA $A = (\Sigma, S, M, S_0, F, \gamma, K)$ and $B = (K, T, N, T_0, G, \lambda, \Sigma)$ is the following process: Let $P_0 = \{(s, t) \mid s \in S_0, t \in T_0\}$ be the initial pair set. If the pair set at stage i is P_i , the next pair set is $P_{i+1} = \{(s, t) \mid s \in M(s', \lambda(t')), t \in N(t', \gamma(s')), (s', t') \in P_i\}$.

We say that the interaction "halts" and A "accepts" B if $P_i \cap F \times T \neq \emptyset$ for some i .

Definition 3. Given an NDLA $A = (S, M, S_0, F, \gamma)$, the characteristic transition function \tilde{M} of A is defined as follows:

$$\tilde{M}: S \times (\bar{K} \Sigma \bar{K})^* \rightarrow 2^S \text{ where } \bar{K} = K \cup \{\epsilon\}.$$

For $\forall s \in S, \forall l$ and $\forall l' \in \bar{K}, \forall \sigma \in \Sigma$ and $\forall x \in (\bar{K} \Sigma \bar{K})^*$,

$$\tilde{M}(s, \Lambda) = s,$$

$$\tilde{M}(s, \sigma l') = \{s' \mid s' \in M(s, \sigma), l \in \{\gamma(s), \epsilon\}, l' \in \{\gamma(s'), \epsilon\}\},$$

$$\tilde{M}(s, x l l') = \bigcup_{s'' \in \tilde{M}(s, x)} \tilde{M}(s'', l l').$$

It is convenient to extend the domain of \tilde{M} to $2^S \times (\bar{K} \Sigma \bar{K})^*$ by $\tilde{M}(R, x) = \bigcup_{s \in R} \tilde{M}(s, x)$ for each $R \in 2^S$ and $x \in (\bar{K} \Sigma \bar{K})^*$.

Definition 4. We define $\text{bh}^+(A)$ and $\text{bh}^-(A)$ such that $\text{bh}^+(A) = \{x \in (K \Sigma K)^* \mid K \Sigma \epsilon \epsilon \Sigma \epsilon \cup \epsilon \Sigma \epsilon \cup \Lambda\} \mid \tilde{M}(S_0, x) \cap F \neq \emptyset\}$, $\text{bh}^-(A) = \{x \in (K \Sigma K)^* \mid \tilde{M}(S_0, x) \neq \emptyset\}$, and $\text{bh}(A) = \{x \in (K \Sigma K)^* \mid \tilde{M}(S_0, x) \cap F \neq \emptyset\}$.

A local automaton has both the behavior of accepting or rejecting other local automata and the behavior of being accepted or rejected by other local automata. The former is $\text{bh}^+(A)$ and the latter is $\text{bh}^-(A)$.

Definition 5. A and A' are equivalent iff $\text{bh}^+(A) = \text{bh}^+(A')$ and $\text{bh}^-(A) = \text{bh}^-(A')$.

Lemma 1: A and A' are equivalent iff $\text{bh}(A) = \text{bh}(A')$ and $\text{bh}^-(A) = \text{bh}^-(A')$.

Proof: To prove the sufficiency we have only to show that $x \in \text{bh}^+(A')$ for every $x \in \text{bh}^+(A)$.

1) If $x = \Lambda$, then $\Lambda \in \text{bh}^+(A')$ implies $\tilde{M}(S_0, \Lambda) \cap F \neq \emptyset$. $\tilde{M}(S_0, \Lambda) \cap F \neq \emptyset \Leftrightarrow \text{bh}(A) = \text{bh}(A') \Leftrightarrow \tilde{M}(S_0, \Lambda) \cap F \neq \emptyset \Leftrightarrow \Lambda \in \text{bh}^+(A)$.

2) If $x \in \epsilon \Sigma \epsilon$, i.e., $x = \epsilon \sigma \epsilon$ for some $\sigma \in \Sigma$, then $x \in \text{bh}^+(A)$ implies $\tilde{M}(S_0, x) \cap F \neq \emptyset$. This implies $M(S_0, \sigma) \cap F \neq \emptyset$ implies $M(s, \sigma) \in s'$ for some $s \in S_0$ and some $s' \in F$. Hence $\tilde{M}(S_0, \gamma(s) \sigma \gamma(s')) \cap F \neq \emptyset$, and this implies $\gamma(s) \sigma \gamma(s') = y \in \text{bh}(A) = \text{bh}(A')$ implies $\tilde{M}(S_0, y) \cap F \neq \emptyset$.

'iff' denotes 'if and only if'.

implies $M'(S_0', \sigma) \cap F' \neq \emptyset$. Hence $\tilde{M}'(S_0', \epsilon \sigma \epsilon) \cap F' \neq \emptyset$, and then this implies $x \in bh^+(A')$.

3) If $x \in (K\Sigma K)^* K\Sigma \epsilon \epsilon \Sigma \epsilon$, we can show that $x \in bh^+(A')$ similarly.

To prove the necessity, it is sufficient to show that $bh^+(A) \neq bh^+(A')$ or $bh^-(A) \neq bh^-(A')$ if $bh(A) \neq bh(A')$. If $bh(A) \neq bh(A')$, then there exists $x \in (K\Sigma K)^*$ such that $x \in bh(A)$ and $x \notin bh(A')$.

1) If $x = \Lambda$, then $\Lambda \notin bh(A')$ implies $\Lambda \notin bh^+(A')$ and $\Lambda \notin bh^-(A)$, hence $bh^+(A) \neq bh^+(A')$.

2) If $x \in K\Sigma K$, then $x = \text{lol}' \in bh(A)$ implies $\epsilon \sigma \epsilon \in bh^+(A)$ and $\text{lol}' \in bh^-(A)$, and $\text{lol}' \notin bh(A')$ implies $\epsilon \sigma \epsilon \notin bh^+(A')$, or $\epsilon \sigma \epsilon \in bh^+(A')$ and $\text{lol}' \notin bh^-(A')$. Since $\epsilon \sigma \epsilon \in bh^+(A)$, $\epsilon \sigma \epsilon \notin bh^+(A')$ implies $bh^+(A) \neq bh^+(A')$. Since $\text{lol}' \in bh^-(A)$, $\epsilon \sigma \epsilon \in bh^+(A')$ and $\text{lol}' \notin bh^-(A')$ imply $bh^-(A) \neq bh^-(A')$. Hence $x \in bh(A)$ and $x \notin bh(A')$ imply $bh^+(A) \neq bh^+(A')$ or $bh^-(A) \neq bh^-(A')$.

3) If $x \in (K\Sigma K)^* K\Sigma K K\Sigma K$, then we can similarly show that $bh^+(A) \neq bh^+(A')$ or $bh^-(A) \neq bh^-(A')$.

3. Homomorphism of CDLA

Definition 6. A local automaton is a "characteristically deterministic" local automaton (CDLA) if $\gamma(s_{0i}) \neq \gamma(s_{0j})$ for $i \neq j$, $\forall s_{0i}$ and $\forall s_{0j} \in S_0$, and $|\tilde{M}(s, \mu)| \leq 1$ for $\forall \mu \in K\Sigma K$ and $\forall s \in S$.

Definition 7. Let $A = (S, M, S_0, F, \gamma)$ and $B = (T, N, T_0, G, \lambda)$ be CDLA's. $\psi: S \rightarrow T$ is a homomorphism of A onto B iff

- 1) $\psi(S_0) = T_0$,
- 2) $\psi(\tilde{M}(s, \mu)) = \tilde{N}(\psi(s), \mu)$ for $\forall \mu \in K\Sigma K$ and $\forall s \in S$,
- 3) $s \in F \Leftrightarrow \psi(s) \in G$,
- 4) $\gamma(s) = \lambda(\psi(s))$.

Definition 8. Given a CDLA $A = (S, M, S_0, F, \gamma)$, two states of A , s_i and s_j , are said to be weakly equivalent, written $s_i \sim s_j$, iff $SC_A(s_i) = SC_A(s_j)$, where $SC_A(s)$, where $SC_A(s) = \{x \in (K\Sigma K)^* \mid \tilde{M}(s, x) \in F\}$.

Definition 9. Let $A = (S, M, S_0, F, \gamma)$ be a CDLA. The quotient A/\sim of A is defined as a 5-tuple $\hat{A} = (\hat{S}, \hat{M}, \hat{S}_0, \hat{F}, \hat{\gamma})$ such that $\hat{S} = S/\sim = \{[s] \mid s \in S\}$, where $[s]$ is the equivalence class of the states of A containing s , $\hat{M}([s], \mu) = [s']$ iff $\tilde{M}(s, \mu) = s'$, $\hat{S}_0 = \{[s] \mid s \in S_0\}$, $\hat{F} = \{[s] \mid s \in F\}$, and $\hat{\gamma}([s]) = \gamma(s)$ for $\forall \mu \in K\Sigma K$, $\forall s$, and $\forall s' \in S$.

Theorem 1: The quotient CDLA A/\sim constructed above is a homomorphic image of any CDLA B which is weakly equivalent to A .

Proof: Let $A/\sim = \hat{A} = (\hat{S}, \hat{M}, \hat{S}_0, \hat{F}, \hat{\gamma})$, $B = (T, N, T_0, G, \lambda)$. Define $\psi: T \rightarrow S$ such that i) for each $t_{0i} \in T_0$ with some $\hat{s}_{0j} \in \hat{S}_0$, $\psi(t_{0i}) = \hat{s}_{0j}$ iff $\lambda(t_{0i}) = \gamma(\hat{s}_{0j})$, and ii) for each $t \in T - T_0$ with some $\hat{s} \in \hat{S}$, $\psi(t) = \hat{s}$ iff $\tilde{N}(T_0, \mu) = t$ and $\tilde{M}(\hat{S}_0, \mu) = \hat{s}$ for some $\mu \in (K\Sigma K)^*$.

- 1) It follows from this definition and Definition 6 that $\psi(T_0) = \hat{S}_0$.
- 2) We can also show that ψ is a mapping of T onto \hat{S} .
 For $\forall \xi \in K\Sigma K$ and $\forall t \in T$, we can write $\psi(t) = \hat{s}$, $\psi(t') = \hat{s}'$ and $\tilde{N}(t, \xi) = t'$. From the definition of ψ , $\tilde{N}(T_0, \mu) = t$, $\tilde{M}(\hat{S}_0, \mu) = \hat{s}$, $\tilde{N}(T_0, \nu) = t'$, and $\tilde{M}(\hat{S}_0, \nu) = \hat{s}'$ for some μ and $\nu \in (K\Sigma K)^*$, and we have $\psi(\tilde{N}(t, \xi)) = \psi(t') = \hat{s}' = \tilde{M}(\hat{S}_0, \nu)$ and $\tilde{M}(\hat{S}_0, \mu\xi) = \tilde{M}(\tilde{M}(\hat{S}_0, \mu), \xi) = \tilde{M}(\psi(t), \xi)$. Since $\tilde{N}(T_0, \nu) = \tilde{N}(T_0, \mu\xi)$, $\tilde{M}(\hat{S}_0, \nu) = \tilde{M}(\hat{S}_0, \mu\xi)$. Hence $\psi(\tilde{N}(t, \xi)) = \tilde{M}(\psi(t), \xi)$ for each $t \in T$ and each $\xi \in K\Sigma K$.
- 3) Let $\psi(t) = \hat{s}$, then $\hat{s} = \tilde{M}(\hat{S}_0, \mu)$ and $t = \tilde{N}(T_0, \mu)$ for some $\mu \in (K\Sigma K)^*$. Hence, $\hat{s} = \psi(t) \in \hat{F} \Leftrightarrow \mu \in \text{bh}(B) = \text{bh}(A) \Leftrightarrow t = \tilde{N}(T_0, \mu) \in G$.
- 4) We can see from the definition of ψ and CDLA that $\lambda(t) = \hat{\gamma}(\psi(t))$ for each $t \in T$.
 By Theorem 1, we have the following corollary.

Corollary 1: A/\sim is the unique (up to isomorphism) CDLA which has the smallest number of states and is equivalent to A .

4. Minimization Algorithm

Given a finite local automaton, we are to construct an NDLA with the same behavior having the fewest states. For ordinary automata, Kameda and Weiner [2] have investigated the minimization problem. In this section, we modify their definitions and algorithm, and apply them to local automata.

Definition 10. The duals \hat{A} and \bar{A} of a local automaton $A = (S, M, S_0, F, \gamma)$ are defined as $\hat{A} = (S, \hat{M}, F, S_0, \gamma)$ and $\bar{A} = (S, \bar{M}, S, S_0, \gamma)$ where \hat{M} is a function such that for $\forall \sigma \in \Sigma$, $\forall s_i$, and $\forall s_j \in S$, $s_j \in \hat{M}(s_i, \sigma) \Leftrightarrow s_j \in M(s_i, \sigma)$.

It is obvious that $\hat{\hat{A}} = \bar{\bar{A}} = A$.

Definition 11. Given an NDLA $A = (S, M, S_0, F, \gamma)$, the succeeding events, $SC_A(s_i)$ and $SC_{\bar{A}}(s_i)$, and preceding event $PR_A(s_i)$ of a state s_i of A are defined by

$$SC_A(s_i) = \text{bh}((S, M, s_i, F, \gamma))$$

$$SC_{\bar{A}}(s_i) = \text{bh}^-(S, M, s_i, F, \gamma)$$

$$PR_A(s_i) = \text{bh}((S, M, S_0, s_i, \gamma)).$$

Two states of A , s_i and s_j , are said to be equivalent, written $s_i \approx s_j$, iff $SC_A(s_i) = SC_A(s_j)$ and $SC_{\bar{A}}(s_i) = SC_{\bar{A}}(s_j)$.

Lemma 2: A CDLA is reduced iff no two distinct states are equivalent.

This proof is immediate from Corollary 1.

Definition 12. Given an NDLA $A = (S, M, S_0, F, \gamma)$, we define the subset CDLA associated with A to be $D(A) = (P, M', P_0, F', \gamma')$, where

$$P_0 = S_0 / \gamma^{-1} \gamma$$

$$P = \{ \tilde{M}(p, x) \mid p \in P_0, x \in (K\Sigma K)^* \} = \{ p_1, p_2, \dots, p_m \}$$

$$F' = \{ p \in P \mid p \cap F \neq \emptyset \}$$

$$M'(p_i, \sigma) = p_j \Leftrightarrow M(p_i, \sigma) / \gamma^{-1} \gamma \ni p_j \text{ for all } i, j, \text{ and } \forall \sigma \in \Sigma.$$

$$\gamma'(p_i) = \gamma(s) \text{ for } \forall s \in p_i.$$

Lemma 3: $\text{bh}(D(A)) = \text{bh}(A)$, $\text{bh}^-(D(A)) = \text{bh}^-(A)$.

This lemma is so easily verified that we omit the proof.

Definition 13. Given an NDLA $A = (S, M, S_0, F, \gamma)$, let $B = D(A) = (P, M', P_0, F', \gamma')$, $C = D(\bar{A}) = (Q, M'', Q_0, F'', \gamma'')$, and $D = D(\bar{\bar{A}}) = (R, M''', R_0''', F''', \gamma''')$. Also we let $s \in S$, $p_i \in P$, $q_j \in Q$, and $r_k \in R$. A states map (SM) of A contains a row for each nonempty state of B , and a column for each nonempty state of C and D . The (i, j) entry contains $p_i \cap q_j$, or is blank if $p_i \cap q_j = \emptyset$. The $(i, m+k)$ entry contains $p_i \cap r_k$, or is blank if $p_i \cap r_k = \emptyset$, where m is the number of elements in Q . An *elementary local automaton matrix* (ELM) of A is obtained from an SM of A by replacing each nonblank entry by 1.

Definition 14. A *reduced local automaton matrix* (RLM) of A is obtained from an ELM of A by merging all the equivalent rows and columns, where merging of two rows (columns) means the replacing of two rows (columns) by a new row (column).

Theorem 2: Let I be an equivalent class of NDLA. A unique (within permutation of the rows and columns) RLM exists for all local automata in I .

This can be proved similarly as done by Kameda [2].

Definition 15. Given an RLM, if all the entries at the intersections of a set of rows $\{p_{i1}, \dots, p_{ia}\}$ and a set of columns $\{q_{j1}, \dots, q_{jb}; r_{k1}, \dots, r_{kc}\}$ are 1's, then this set of 1's is said to form a grid. We represent the grid by $g = \{p_{i1}, \dots, p_{ia}; q_{j1}, \dots, q_{jb}; r_{k1}, \dots, r_{kc}\}$.

Definition 16. A set of grids forms a cover if every 1 in the RLM belongs to at least one grid in the set.

Definition 17. Let $A = (P, M, P_0, F, \gamma)$ be a CDLA. The pair $\langle Z, f \rangle$ is called a subset assignment to B if Z is a finite set and $f: P \rightarrow 2^Z - \{\emptyset\}$ is a function. The natural subset assignment to the subset CDLA $D(A)$ is $\langle S, f \rangle$, where S is the set of states of A and $f(p) = \{s \mid s \in P\}$ for $\forall p \in P$.

Definition 18. Let $B = (P, M, P_0, F, \gamma)$ be a CDLA, and let $\langle Z, f \rangle$ be a subset assignment to B . Then define an NDLA $I(Z, f, B) = (Z, N, Z_0, G, \gamma)$, where for $\forall z \in Z$, $\forall p \in P$, and $\forall \mu \in K\Sigma K$,

- 1) $Z_0 = f(P_0)$
- 2) $z \in G \Leftrightarrow [z \in f(p) \Leftrightarrow p \in F]$
- 3) $z' \in \tilde{N}(z, \mu) \Leftrightarrow [z \in f(p) \Rightarrow z' \in f(\tilde{M}(p, \mu))]$
- 4) $\lambda(z) = \gamma(p)$ for $z \in f(p)$.

$I(Z, f, B)$ is called the NDLA obtained by the intersection rule from B .

Definition 19. A subset assignment $\langle Z, f \rangle$ to a CDLA B is legitimate iff $\text{bh}(I(Z, f, B)) = \text{bh}(B)$ and $\text{bh}^{-1}(I(Z, f, B)) = \text{bh}^{-1}(B)$.

Lemma 4: For $\langle Z, f \rangle$, $\text{bh}(I(Z, f, B)) \subseteq \text{bh}(B)$ and $\text{bh}^{-1}(I(Z, f, B)) \subseteq \text{bh}^{-1}(B)$.

We omit the proof (see [2]).

Lemma 5: The natural subset assignment is legitimate.

Proof: Let $A = (S, M', S_0, F', \gamma')$ and $B = D(A) = (P, M, P_0, F, \gamma)$. We show that $\text{bh}(I(S, f, B)) = \text{bh}(B)$ and $\text{bh}^{-1}(I(S, f, B)) = \text{bh}^{-1}(B)$ for the natural subset assignment $\langle S, f \rangle$. Let $I(S, f, B) = (S, N, Z_0, G, \gamma)$. Then for $\forall \sigma \in \Sigma$, $\forall s \in S$, $\forall p \in P$ and $\forall \mu \in K\Sigma K$,

$$1) Z_0 = P_0 = S_0$$

$$2) s \in F' \Rightarrow [s \in P \Rightarrow p \in F] \Rightarrow s \in G, \text{ so that } F' \subseteq G.$$

$$3) s' \in \tilde{M}'(s, \mu) \Rightarrow [s \in P \Rightarrow s' \in \tilde{M}(p, \mu)] \Rightarrow s' \in \tilde{N}(s, \mu), \text{ so that } \tilde{M}'(s, \mu) \subseteq \tilde{N}(s, \mu).$$

Thus it is clear that $\tilde{M}'(S_0, x) \cap F' \neq \emptyset \Rightarrow \tilde{N}(Z_0, x) \cap G \neq \emptyset$ and $\tilde{M}'(S_0, x) \neq \emptyset \Rightarrow \tilde{N}(Z_0, x) \neq \emptyset$. These imply that $\text{bh}(A) \subseteq \text{bh}(I(S, f, B))$ and $\text{bh}^{-1}(A) \subseteq \text{bh}^{-1}(I(S, f, B))$. But $\text{bh}(A) = \text{bh}(B)$ and $\text{bh}^{-1}(A) = \text{bh}^{-1}(B)$ by Lemma 3 and $\text{bh}(I(S, f, B)) \subseteq \text{bh}(B)$ and $\text{bh}^{-1}(I(S, f, B)) \subseteq \text{bh}^{-1}(B)$ by Lemma 4. Hence $\text{bh}(I(S, f, B)) = \text{bh}(B)$ and $\text{bh}^{-1}(I(S, f, B)) = \text{bh}^{-1}(B)$.

We can prove the following theorem just the same way as Kameda [2].

Theorem 3: Given any NDLA A , there exists a legitimate cover over an RLM of A such that the NDLA obtained by the intersection rule using the subset assignment associated with the cover is a minimum NDLA.

With these preparations, we are now to describe a minimization algorithm of NDLA's.

Minimization Algorithm

- 1) Given an NDLA, construct its RLM and find a minimum cover.
- 2) Let i_0 be the number of grids in the minimum cover derived above and set $i = i_0$.
 - a) For each cover containing i grids, test whether it is legitimate.
 - b) If no legitimate cover is found, set $i = i + 1$, and go to step a).

It is obvious that the process should terminate after a finite number of cycles.

5. Conclusion

In this paper we have described a procedure for finding a minimum state nondeterministic finite local automaton equivalent to a given finite local automaton. The minimization theory for local automata can be given in almost the same way as the standard theory by introducing the characteristic transition function and the class of CDLA's.

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