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Optimal Granularity of Parallel Test Generation on the Client-Agent-Server Model

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This paper proposes a Client-Agent-Server model (CAS model) which can decrease the work load of the client by adding agent processors to the Client-Server model and presents an approach to parallel test generation for logic circuits on the CAS model. In this paper, we consider the fault parallelism in which a cluster of faults will be allocated from the client processor to an agent processor and from an agent processor to a server processor for the CAS model. Hence, we have to consider two granularities; one is the size of the cluster between the client and agents, and the other is the size of the cluster between agents and servers. We formulate the problem of test generation for the CAS model and analyze the optimal pair of granularities in both cases of *static* and *dynamic* task allocation. Finally, we present experimental results based on an implementation of our CAS model on a network of workstations using the ISCAS'89 benchmark circuits. The experimental results are very close to the analytical results which confirms the existence of an optimal pair of granularities that minimizes the total processing time for benchmark circuits as well as analysis.

1. Introduction

Theoretically, it is shown that the problem of test generation for logic circuits is NP-hard[1],[2] even for combinational circuits, and hence it is very difficult to speed up the test generation process due to backtracking mechanism. On the other hand, efficient heuristics to speed up test generation have been proposed[3]-[5] but handling the increased logic complexity of VLSI circuits has been severely limited by the slowness of conventional CAD tools on a general purpose computer. Multiprocessing hardware has to be used to get orders of magnitude speed up for those circuits of VLSI or ULSI complexity.

There are several types of parallelism inherent in test-pattern generation: fault parallelism, search parallelism, heuristic parallelism and topological parallelism.[14] *Fault parallelism* refers to dealing with different faults in parallel. Motohara et al.,[7] Patil and Banerjee,[12] and Fujiwara and Inoue[10] presented their methods of parallel processing for test generation based on fault parallelism. *Search parallelism* refers to searching different nodes of a decision tree (in a branch-and-bound search) or to searching different input-vectors in parallel. Motohara et al.[7] and Patil and Banerjee[11] proposed their methods of parallel processing for test genera-

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tion based on search parallelism. *Heuristic parallelism* refers to dealing with one fault using different heuristics in parallel. Chandra and Patel[8] reported an approach to heuristic parallelism. *Topological parallelism* refers to simulating different sub-circuits in parallel. Kramer[6] and Hirose et al.[9] presented their methods of parallel processing for topological parallelism.

In Ref. 10), we presented an approach to parallel test generation based on fault parallelism in a loosely-coupled distributed network of general purpose computers and analyzed theoretically the effect of the allocation of target faults to processors using *a Client-Server model (CS model)* illustrated in **Fig. 1**. We showed the existence of the optimal granularity or the optimal number of target faults allocated to processors which minimizes the total processing time for the CS model. For this CS model, as the number of processors increases, communication overhead among processors also increases, and hence, the total performance goes down. This problem of performance degradation can be usually resolved by using a hierarchical approach.

In this paper, as the hierarchical approach, we propose *a Client-Agent-Server model (CAS model)* which can decrease the work load of the client by adding agent processors to the CS model. We consider the fault parallelism in which a cluster of faults will be allocated from
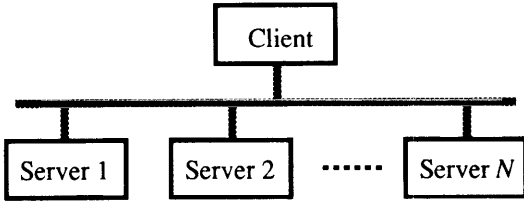
Fig. 1   Architecture of the Client-Server model.

the client processor to an agent processor and from an agent processor to a server processor for the CAS model. Hence, we have to consider two granularities; one is the size of the cluster between the client and agents, and the other is the size of the cluster between agents and servers. We formulate the problem of test generation for the CAS model and analyze the optimal pair of granularities in both cases of *static* and *dynamic* task allocation. Finally, we present experimental results based on an implementation of our CAS model on a network of workstations using the ISCAS'89 benchmark circuits. The experimental results are very close to the analytical results which confirms the existence of an optimal pair of granularities that minimizes the total processing time for benchmark circuits as well as analysis.

2. Architecture of the Client-Agent-Server Model

The architecture of our loosely-coupled multiple processor systems is illustrated in **Fig. 2**. This system is derived by inserting *agent* processors between a client and servers of the CS model. We call it a *Client-Agent-Server model* (*CAS model*). In this CAS model, N_a agents are connected to the client, and N_s servers are connected to each agent, where all processors are connected to a single communication network.

The client requests an agent to execute a task and to return the result. An agent partitions a task into sub-tasks and distributes each sub-task to a server connected to the agent. When a server finishes its assigned task, it sends the result to the agent and requests a new task. After an agent finishes the task from the client, it sends the result to the client and requests a new task. The client saves the result, and sends a new task to the agent. This process is repeated until all tasks are processed.

Here if we regard the task as test generation for faults in a given circuit, the above process can be redescribed as follows:

The client first generates a fault table of the faults. The client extracts a number of faults from the fault table as a set of target faults, and sends the faults to an agent. When an agent receives the target faults from the client, the agent sends a subset of the target faults to a server connected to the agent as a set of target faults for the server. A server which received the target faults generates a test-pattern for one of the target faults, and finds out all detected faults by the test-pattern by performing simulation for all faults in the circuit, not just those in the set of target faults. The server repeats test-pattern generation and fault simulation for all the target faults, and then sends the result to the agent. After receiving the result from the server, the agent saves it in its own storage. The agent then sends a new set of target faults which have not yet been processed by any server of the agent, and sends it to the server. After all the target faults assigned to the agent are processed, the agent sends the results to the client and requests a new set of target faults. The client updates the fault table, and sends new target faults to the agent. This process continues until all faults in
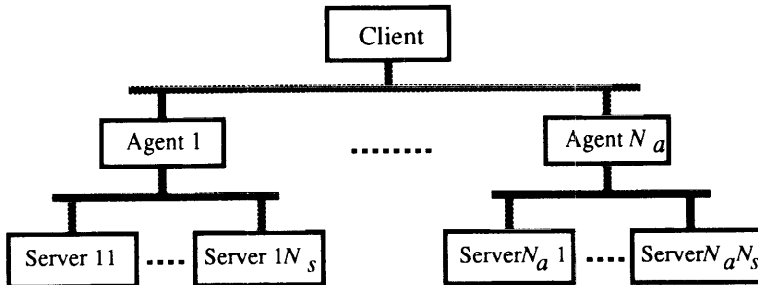

Fig. 2   Architecture of the Client-Agent-Server model.

the fault table are processed.

3. Formulation of the Problem

We formulate the test generation problem for the CAS model. It consists of one client, N_a agents and N_s servers per agent. Let the k-th server connected to the j-th agent A_j be server S_{jk}. A process of test-pattern generation for a fault f_i is called *a process for fault* f_i. The result of a process for a fault is whether 1) the fault is detected by a test-pattern, or 2) the fault is redundant, or 3) the process is aborted due to the exceeded backtracking.

The parameters used here are defined as follows:

M: the total number of faults of a given circuit.

τ_{ijk}: the processing time of server S_{jk} for fault f_i.

δ_{ijk}: the probability that process for fault f_i is allocated to server S_{jk}.

λ_{aij}: the probability that agent A_j communicates to the client after process for fault f_i.

λ_{sijk}: the probability that server S_{jk} communicates to Agent A_j after process for fault f_i.

τ_{ca}: the mean communication time which includes waiting time due to contention and data transfer time between the client and agents.

τ_{cs}: the mean communication time which includes waiting time due to contention and data transfer time between an agent A_j and servers.

Then, the average time necessary to complete all processes allocated to server S_{jk} is

$$T_{jk} = \sum_{i=1}^{M} \delta_{ijk} (\tau_{ijk} + \lambda_{aij}\tau_{ca} + \lambda_{sijk}\tau_{cs}). \quad (1)$$

The time necessary to complete all processes is defined by the maximum of T_{jk}:

$$T = \max\{T_{jk}\}. \quad (2)$$

4. Optimal Granularity with Static Task Allocation

First we consider *static* task allocation of faults where the numbers of target faults from the client to an agent and from an agent to a server are always constant respectively.

4.1 Assumption of Homogeneous Problem

To obtain the minimum processing time on the CAS model, it is important to equalize the load of each server. Here, we shall assume a *homogeneous* case is follows:

(1) All servers are uniform, i.e.,

$$\tau_{ijk} = \tau_i \quad (3)$$

for all faults f_i and servers S_{jk}.

(2) For any fault f_i, the probability that fault f_i is allocated to a server S_{jk} is independent of the server S_{jk}, i.e.,

$$\delta_{ijk} = \delta_i \quad (4)$$

for all faults f_i and servers S_{jk}.

4.2 Communication Probability: λ_{aij}, λ_{sijk}

Let m_a be the number of target faults transferred from the client to an agent A_j during each communication. Suppose that fault f_i is in the set of m_a target faults allocated to the agent A_j. Then the probability that the agent A_j communicates to the client after process for fault f_i is

$$\lambda_{aij} = \frac{1}{m_a} \quad (5)$$

since such a communication occurs only once for those m_a faults.

Let m_s be the number of target faults transferred from an agent A_j to a server S_{jk} during each communication. Suppose that fault f_i is in the set of m_s target faults allocated to the server S_{jk} from the agent A_j. Then the probability that the server S_{jk} communicates to the agent A_j after process for fault f_i is

$$\lambda_{sijk} = \frac{1}{m_s} \quad (6)$$

since such a communication occurs only once for those m_s faults.

4.3 Probability of Process Allocation: δ_{ijk}

Suppose that the client requests an agent to process m_a target faults. The agent extract m_s faults from the m_a target faults, and requests a server to process the m_s target faults. Note that $m_s \le m_a$. The server generates a test-pattern for one of the m_s faults, and find out all the faults detected by the test-pattern by performing fault simulation for all faults, not just those in the set of m_s target faults. It repeats test-pattern generation and fault simulation until all target faults are processed. Let ρm_s be the number of faults that are newly detected or found to be redundant at completion of test generation for m_s target faults. Let us call those faults *newly processed faults*.

Let us define the ratio of newly processed faults to target faults:

number of newly processed faults

$$\rho = \frac{\text{per server } (\rho m_s)}{\text{number of target faults per server } (m_s)} \quad (7)$$

Note that this ratio will decrease as the number of processed faults increases. Therefore, it is expressed as ρ_i, the ratio for ith processed fault f_i.

During each iteration of the server process, m_s target faults are processed by the server and ρm_s faults are either detected or identified to be redundant through both test-pattern generation and fault simulation. Hence, the probability that fault f_i is allocated as a target fault to some server is

$$\frac{m_s}{\rho_i m_s} = \frac{1}{\rho_i}. \quad (8)$$

On the other hand, the probability that the process for fault f_i is allocated to some server is defined by

$$\sum_{j=1}^{N_a} \sum_{k=1}^{N_s} \delta_{ijk}. \quad (9)$$

Therefore, we have

$$\sum_{j=1}^{N_a} \sum_{k=1}^{N_s} \delta_{ijk} = \frac{1}{\rho_i}. \quad (10)$$

From the assumption that $\delta_{ijk} = \delta_i$, we have

$$\sum_{j=1}^{N_a} \sum_{k=1}^{N_s} \delta_{ijk} = \sum_{j=1}^{N_a} \sum_{k=1}^{N_s} \delta_i = N_a N_s \delta_i. \quad (11)$$

Hence, we have

$$\delta_{ijk} = \delta_i = \frac{1}{N_a N_s \rho_i}. \quad (12)$$

4.4 Ratio of Newly Processed Faults to Target Faults: ρ

The number of newly processed faults will quickly decrease as the number of processed faults increases. Further, the number of newly processed faults per fault will decrease as the number of target faults per server and the number of servers increase. In Ref. 10), we assumed the ratio of newly processed faults to target faults for the CS model to be

$$\rho(x) = \frac{1}{r_0 + r_1 x + r_2 mN} \quad (13)$$

where m is the number of target faults to a server per communication, N is the number of servers, x is the number of processed faults and r_0, r_1 and r_2 are constants. In this expression, the factor $1/(r_0 + r_1 x)$ expresses the effect of fault simulation, and the factor $r_2 mN$ accounts for the decrease ratio of newly processed faults due to over-lapped processing (see Ref. 10)).

About the factor for decrease ratio of newly processed faults on the CAS model, we have to consider the overlapped processing among agents, in addition to the overlapped processing among servers. After receiving the list of the result from a server, an agent renews its own fault table, which is the copy from the client. Since multiple agents are working simultaneously, some agents may save the same faults detected by servers. These overlapped processes will increase and hence ρ_i will decrease as the number of target faults per agent (m_a), and the number of agents (N_a) increase. By introducing this factor $(m_a N_a)$ into the expression (13), we have

$$\rho(x) = \frac{1}{r_0 + r_1 x + r_2 m_s N_s + r_3 m_a N_a} \quad (14)$$

where r_0, r_1, r_2 and r_3 are constants. In the above expression, the factor $r_3 m_a N_a$ accounts for the decrease ratio due to the overlapped processing among agents.

4.5 Communication Time: τ_{ca}, τ_{cs}

Here we have the following assumptions:
1) The size of data (fault table) transferred between the client and an agent, or between an agent and a server is fixed, and hence, the data transfer time during communication between the client and agents, or between an agent and servers is a constant.
2) All agents communicate with the client through a single communication network. All servers also communicate with respective agents through the same network. Agents and servers can not consequently communicate while one of the other processors communicates. Hence, the waiting time during communication between the client and an agent, or between an agent and a server is proportional to the number of agents plus the total number of servers, i.e., $N_a + N_a N_s$.
3) After receiving the result from an agent, the client updates the fault table, and sends a new set of target faults. This work load increases in proportion to the number of agents, N_a. Hence, the waiting time during working of the client is proportional to N_a. On the other hand, the work load of an agent increases in proportion to the number of the servers connected to the agent, N_s. Hence, the waiting time during working of an agent is proportional to N_s.

From the above assumptions, we have

$$\tau_{ca} = t_{a0} + t_{a1} N_a (N_s + 1) + t_{a2} N_a \quad (15)$$

where t_{a0}, t_{a1} and t_{a2} are constants. And we have

$$\tau_{cs} = t_{s0} + t_{s1}N_a(N_s+1) + t_{s2}N_s \qquad (16)$$

where t_{s0}, t_{s1} and t_{s2} are constants.

Here we assume $t_{a0} = t_{s0} = t_0$, $t_{a1} = t_{s1} = t_1$, and $t_{a2} = t_{s2} = t_2$. Then we have

$$\tau_{ca} = t_0 + t_1 N_a(N_s+1) + t_2 N_a \qquad (17)$$

and

$$\tau_{cs} = t_0 + t_1 N_a(N_s+1) + t_2 N_s \qquad (18)$$

where t_0, t_1 and t_2 are constants.

4.6 Total Processing Time: T

Suppose that the number of processed faults is i when fault $f_{\pi(i)}$ is processed where π is a permutation of $I_M = \{1, 2, \cdots, M\}$. Then, from the expression (14), the ratio of newly processed faults when fault $f_{\pi(i)}$ is processed can be expressed as

$$\rho_{\pi(i)} = \frac{1}{r_0 + r_1 i + r_2 m_s N_s + r_3 m_a N_a}. \qquad (19)$$

Let P be the set of all permutations of I_M. There is a one-to-one correspondence between permutations of I_M and sequences of faults. The total number of sequences is $M!$.

From the expressions (1), (12), (17), (18) and (19), we can derive the average of total processing time for all permutations:

$$\begin{aligned} T = \frac{1}{M!} \sum_{\pi \in P} \sum_{i=1}^{M} \frac{1}{N_a N_s} \\ \cdot (r_0 + r_1 i + r_2 m_s N_s + r_3 m_a N_a) \\ \cdot \left(\tau_i + \frac{\tau_{ca}}{m_a} + \frac{\tau_{cs}}{m_s} \right). \end{aligned} \qquad (20)$$

On the other hand we have

$$\sum_{\pi \in P} \sum_{i=1}^{M} i \tau_{\pi(i)} = \sum_{i=1}^{M} i \left((M-1)! \sum_{i=1}^{M} \tau_i \right). \qquad (21)$$

Substituting the mean processing time for each fault:

$$\tau = \frac{1}{M} \sum_{i=1}^{M} \tau_i \qquad (22)$$

into the right side of the above equation (21), we have

$$\sum_{\pi \in P} \sum_{i=1}^{M} i \tau_{\pi(i)} = \sum_{i=1}^{M} i (M! \tau). \qquad (23)$$

Hence, from (20) and (23) we have

$$\begin{aligned} T = \sum_{i=1}^{M} \frac{1}{N_a N_s} (r_0 + r_1 i + r_2 m_s N_s + r_3 m_a N_a) \\ \cdot \left(\tau + \frac{\tau_{ca}}{m_a} + \frac{\tau_{cs}}{m_s} \right) \end{aligned} \qquad (24)$$

$$= \frac{M}{N_a N_s} \left(r_0 + r_1 \frac{M+1}{2} + r_2 m_s N_s \right. \\ \left. + r_3 m_a N_a \right) \left(\tau + \frac{\tau_{ca}}{m_a} + \frac{\tau_{cs}}{m_s} \right) \qquad (25)$$

Partially differentiating T by m_s, we have

$$\begin{aligned} \frac{\partial T}{\partial m_s} = \frac{M}{N_a N_s} \left(r_2 N_s \left(\tau + \frac{\tau_{ca}}{m_a} \right) \right. \\ \left. - \frac{\left(r_0 + r_1 \frac{M+1}{2} + r_3 m_a N_a \right) \tau_{cs}}{m_s^2} \right) \end{aligned} \qquad (26)$$

Then, we have

$$\begin{aligned} T_{smin} = \frac{M}{N_a N_s} \left(\sqrt{r_2 N_s \tau_{cs}} \right. \\ \left. + \sqrt{\left(r_0 + r_1 \frac{M+1}{2} + r_3 m_a N_a \right) \left(\tau + \frac{\tau_{ca}}{m_a} \right)} \right)^2 \end{aligned} \qquad (27)$$

when

$$m_{sopt} = \sqrt{ \frac{\left(r_0 + r_1 \frac{M+1}{2} + r_3 m_a N_a \right) \tau_{cs}}{r_2 N_s \left(\tau + \frac{\tau_{ca}}{m_a} \right)} } \qquad (28)$$

Partially differentiating T_{smin} by m_a, we have

$$\begin{aligned} \frac{\partial T_{smin}}{\partial m_a} = \frac{M}{N_a N_s} \\ \cdot \left(1 + \sqrt{ \frac{r_2 N_s \tau_{cs}}{\left(r_0 + r_1 \frac{M+1}{2} + r_3 m_a N_a \right) \left(\tau + \frac{\tau_{ca}}{m_a} \right)} } \right) \\ \cdot \left(r_3 N_a \tau - \frac{\left(r_0 + r_1 \frac{M+1}{2} \right) \tau_{ca}}{m_a^2} \right) \end{aligned} \qquad (29)$$

Then, we have the minimum of T

$$\begin{aligned} T_{min} = \frac{M}{N_a N_s} \left(\sqrt{r_2 N_s \tau_{cs}} + \sqrt{r_3 N_a \tau_{ca}} \right. \\ \left. + \sqrt{\left(r_0 + r_1 \frac{M+1}{2} \right) \tau} \right)^2 \end{aligned} \qquad (30)$$

when

$$m_{aopt} = \sqrt{ \frac{\left(r_0 + r_1 \frac{M+1}{2} \right) \tau_{ca}}{r_3 N_a \tau} } \qquad (31)$$

and

$$m_{sopt} = \sqrt{ \frac{\left(r_0 + r_1 \frac{M+1}{2} \right) \tau_{cs}}{r_2 N_s \tau} } \qquad (32)$$

which is derived from the expression (28) by substituting m_a in the expression for m_{aopt}.

Figure 3 shows the graph of the total processing time T as a function of the number of target faults for an agent, m_a, and the number of target faults for a server, m_s. From this figure we can see that there exists an optimal number of target faults for an agent, m_{aopt}, and an optimal number of target faults for a server, m_{sopt}, which minimize the total processing time.

$M = 10000$
$Na = 4$
$Ns = 8$
$\tau = 0.5$
$r_0 = 0.0001$
$r_1 = 0.00001$
$r_2 = 0.0005$
$r_3 = 0.00004$
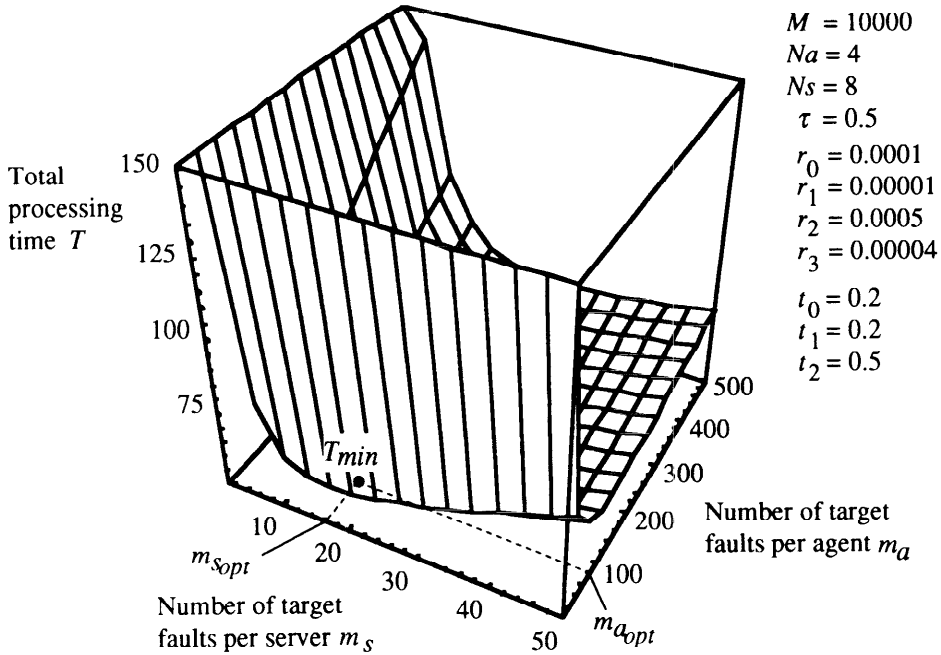
$t_0 = 0.2$
$t_1 = 0.2$
$t_2 = 0.5$

Fig. 3　Total processing time versus granularity : Analysis.

The parallel test generation system of the CAS model was implemented on a network (Ethernet) of workstations (SUN4/LC's). The FAN algorithm[4] was used as a test-pattern generator. **Figures 4, 5** and **6** give the graphs of the total processing time T as a function of the number of target faults for an agent, m_a, and the number of target faults for a server, m_s, for circuits s9234, s13207 and s15850, respectively, of the ISCAS'89 benchmark circuits[13] modified into combinational circuits by assuming full-scan design. In these figures, we can see that the shape of the graphs coincides closely with that of Fig. 3 obtained from the above analysis and hence there exists an optimal granularity pair which minimizes the total processing time.

5. Optimal Granularity with Dynamic Task Allocation

In this section we shall consider *dynamic* task allocation of faults where the numbers of target faults for an agent and for a server will respectively vary as time goes on.

Here, we consider again the homogeneous case; i.e., $\tau_{ijk} = \tau_i$ and $\delta_{ijk} = \delta_i$ for all faults f_i and servers S_{jk}. Suppose that the number of processed faults is i when fault $f_{\pi(i)}$ is processed where π is a permutation of $I_M - \{1, 2, \cdots, M\}$. Let m_{ai} and m_{si} be the numbers of target faults allocated to an agent and a server, respectively, when i faults have been processed by all servers till then. Then the average of total processing time T can be obtained by replacing m_a by m_{ai} and m_s by m_{si} in (20) as follows:

$$T = \frac{1}{M!} \sum_{\pi \in P} \sum_{i=1}^{M} \frac{1}{N_a N_s}$$
$$\cdot (r_0 + r_1 i + r_2 m_{si} N_s + r_3 m_{ai} N_a)$$
$$\cdot \left(\tau + \frac{\tau_{ca}}{m_{ai}} + \frac{\tau_{cs}}{m_{si}} \right) \tag{33}$$

$$= \sum_{i=1}^{M} \frac{1}{N_a N_s} (r_0 + r_1 i + r_2 m_{si} N_s + r_3 m_{ai} N_a)$$
$$\cdot \left(\tau + \frac{\tau_{ca}}{m_{ai}} + \frac{\tau_{cs}}{m_{si}} \right) \tag{34}$$

Partially differentiating the above expression by m_{si}, we have

$$\frac{\partial T}{\partial m_{si}} = \frac{M}{N_a N_s} \left(r_2 N_s \left(\tau + \frac{\tau_{ca}}{m_{ai}} \right) \right.$$
$$\left. - \frac{(r_0 + r_1 i + r_3 m_{ai} N_a) \tau_{cs}}{m_{si}^2} \right) \tag{35}$$

Then, we have

Total processing time T (sec)

Circuit: s9234

$Na = 4$
$Ns = 8$

Number of target faults per agent

Number of target faults per server
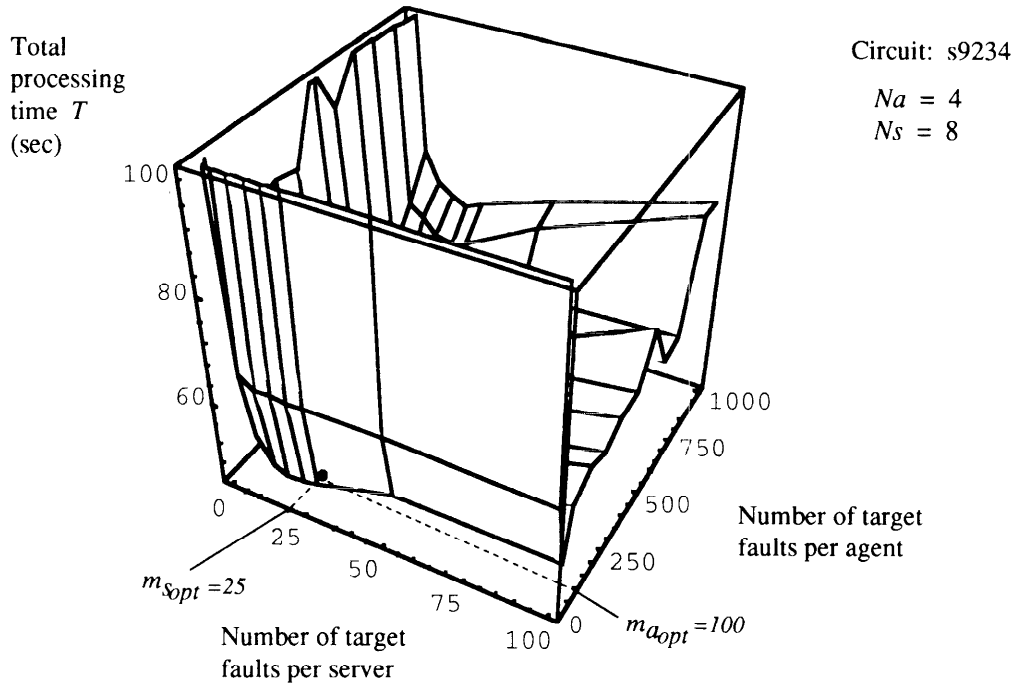
Fig. 4   Total processing time versus granularity : Experimental result for circuit s9234.

Total processing time T (sec)

Circuit: s13207

$Na = 4$
$Ns = 8$

Number of target faults per agent

Number of target faults per server
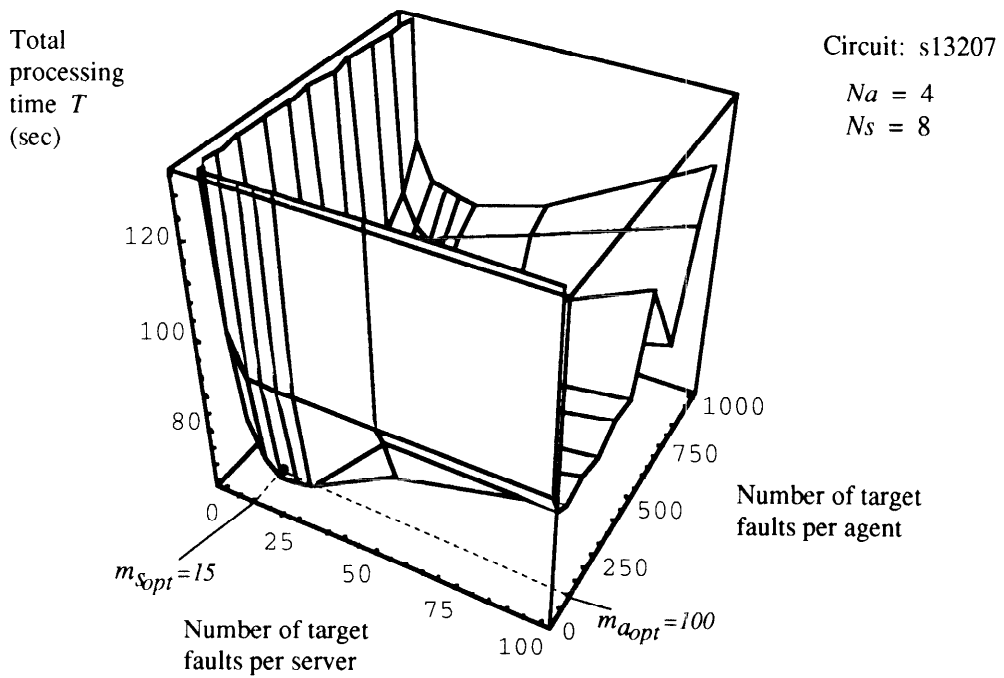
Fig. 5   Total processing time versus granularity : Experimental result for circuit s13207.

Total
processing
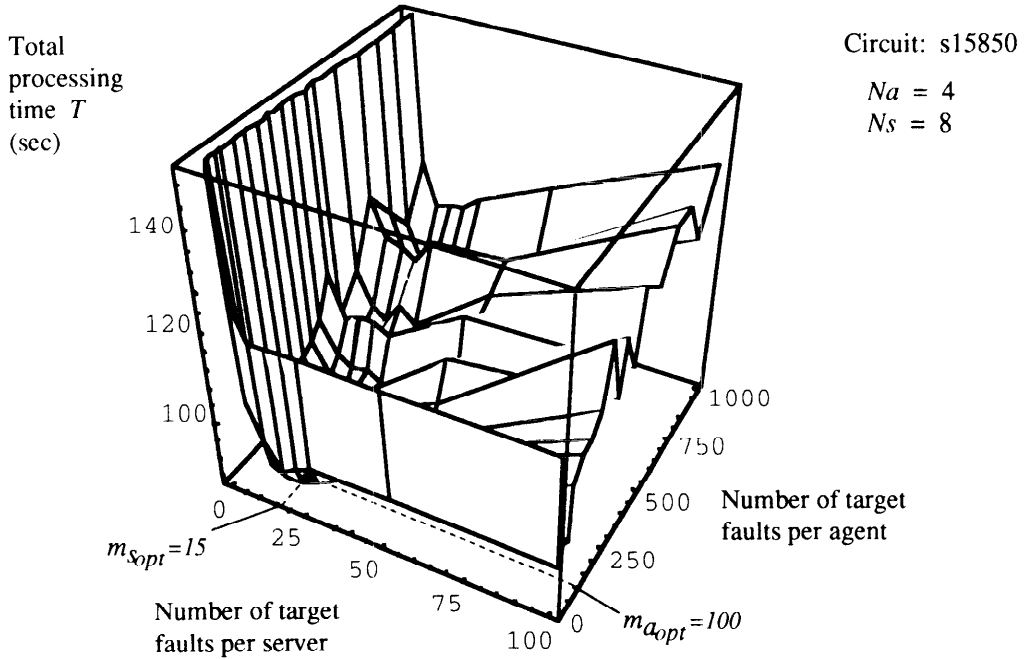time T
(sec)

Circuit: s15850

Na = 4
Ns = 8

Fig. 6   Total processing time versus granularity : Experimental result for circuit s15850.

$$T_{smin} = \sum_{i=1}^{M} \frac{1}{N_a N_s} \left(\sqrt{r_2 N_s \tau_{cs}} \right.$$
$$\left. + \sqrt{(r_0 + r_1 i + r_3 m_a N_a) \left(\tau + \frac{\tau_{ca}}{m_a} \right)} \right)^2 \quad (36)$$

when

$$m_{si} = \sqrt{\frac{(r_0 + r_1 i + r_3 m_a N_a) \tau_{cs}}{r_2 N_s \left(\tau + \frac{\tau_{ca}}{m_a} \right)}} \quad (37)$$

Partially differentiating T_{smin} by m_{ai}, we have

$$\frac{\partial T_{smin}}{\partial m_{ai}} = \frac{M}{N_a N_s}$$
$$\cdot \left(1 + \sqrt{\frac{r_2 N_s \tau_{cs}}{(r_0 + r_1 i + r_3 m_{ai} N_a) \left(\tau + \frac{\tau_{ca}}{m_{ai}} \right)}} \right)$$
$$\cdot \left(r_3 N_a \tau - \frac{(r_0 + r_1 i) \tau_{ca}}{m_{ai}^2} \right) \quad (38)$$

Then, we have the minimum of T for dynamic allocation:

$$T_{dynamic} = \sum_{i=1}^{M} \frac{1}{N_a N_s} (\sqrt{r_2 N_s \tau_{cs}} + \sqrt{r_3 N_a \tau_{ca}}$$
$$+ \sqrt{(r_0 + r_1 i) \tau})^2 \quad (39)$$

when

$$m_{ai} = \sqrt{\frac{(r_0 + r_1 i) \tau_{ca}}{r_3 N_a \tau}} \quad (40)$$

and

$$m_{si} = \sqrt{\frac{(r_0 + r_1 i) \tau_{cs}}{r_2 N_s \tau}}, \quad \text{for all } i. \quad (41)$$

From the above expressions (40) and (41), the optimal granularity (the optimal size of target faults) of time t can be expressed as

$$m_a(t) = \sqrt{\frac{(r_0 + r_1 x_t) \tau_{ca}}{r_3 N_a \tau}} \quad (42)$$

and

$$m_s(t) = \sqrt{\frac{(r_0 + r_1 x_t) \tau_{cs}}{r_2 N_s \tau}} \quad (43)$$

where x_t is the total number of faults processed by all servers till the time t. Hence, the best performance or the test generation with the minimum computation time will be achieved if the dynamic task allocation is scheduled in accordance with the above expression as follows: The client counts up the total number x_t of processed faults till now (at time t), calculates the number $m_a(t)$ of target faults from the equation (42), and then allocates $m_a(t)$ target faults with the number x_t to an agent. The agent calculates the number $m_s(t)$ of target faults from the equation (43), picks the $m_s(t)$ target faults out of the $m_a(t)$ target faults, and then allocates the $m_s(t)$ target faults to an idle server. Note

that although the equations (42) and (43) represent continuous functions, $m_a(t)$ and $m_s(t)$ are respectively defined as integers.

Let us consider next how much reduction of computation time will be achieved by dynamic task allocation compared with static one. The minimum of T for static allocation is

$$T_{\text{static}} = \frac{M}{N_a N_s} \left(\sqrt{r_2 N_s \tau_{cs}} + \sqrt{r_3 N_a \tau_{ca}} \right.$$
$$\left. + \sqrt{\left(r_0 + r_1 \frac{M+1}{2} \right) \tau} \right)^2 \qquad (44)$$

Hence, the difference between T_{static} and T_{dynamic} is

$$T_{\text{static}} - T_{\text{dynamic}}$$
$$= \frac{2\sqrt{\tau} \left(\sqrt{r_2 N_s \tau_{cs}} + \sqrt{r_3 N_a \tau_{ca}} \right)}{N_a N_s}$$
$$\cdot \sum_{i=1}^{M} \left(\sqrt{r_0 + r_1 \frac{M+1}{2}} - \sqrt{r_0 + r_1 i} \right) \quad (45)$$

This equation is always positive for $M > 1$, that is, the dynamic task allocation is always more efficient than the static one.

6. Conclusions

In this paper we presented an approach to parallel processing based on fault parallelism for test generation in a loosely-coupled distributed networks of general-purpose processors. In order to get a more efficient scheme than the CS model, we proposed another model called a Client-Agent-Server model (CAS model) which can decrease the work load of the client by adding agent processors to the CS model.

We considered two granularities; one is the size of the cluster between the client and agents, and the other is the size of the cluster between agents and servers. We formulated the problem of test generation for the CAS model, and analyzed the optimal pair of granularities in both cases of static and dynamic task allocation. We presented experimental results based on an implementation of our CAS model on a network of workstations using the ISCAS'89 benchmark circuits. The experimental results are very close to the analytical results which confirms the existence of an optimal pair of granularities that minimizes the total processing time for benchmark circuits.

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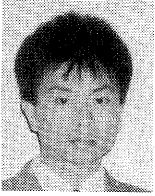
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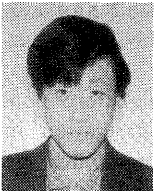
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